# Symmetric Functions

In these exercises, we will use the formula and/or a table of a function to determine if the function is even, odd, or neither.

1. Determine if each of the following functions are even, odd, or neither, *by examining a graph* using your graphing calculator.

   a) $f(x) = x^3 + 6x$ b) $f(x) = |3x| - 7$ c) $f(x) = \dfrac{x}{x+1}$

   d) $f(x) = \dfrac{x}{x^2+1}$ e) $f(x) = 2^x$ f) $f(x) = x^4 - 3x^2 + 13$

2. Determine if each of the following functions are even, odd, or neither, *by checking and simplifying the formula for* $f(-x)$. Remember that if $f(-x) = f(x)$, it means that you have an even function, and if $f(-x) = -f(x)$, it means that you have an odd function.

   a) $f(x) = x^4$ b) $f(x) = |3 - x|$ c) $f(x) = \dfrac{x^2}{x^2+3x+1}$

   d) $f(x) = x^3 \cdot \sqrt{1+x^2}$ e) $f(x) = 2^x$ f) $f(x) = \dfrac{x}{x-1}$

3. Why are the types of symmetry that we are studying called *even* and *odd*? What is the connection to even and odd numbers? Hint: consider a special kind of function called a *power* function. These have formulas of the form $f(x) = x^n$. Take a look at graphs of $y = x^1, y = x^2, y = x^3, \ldots$.

4. There is (only) one function that is *both* even and odd. What is its formula?

5. It's not always clear from a formula whether a function is even or odd. Investigate the function $f$ defined by $f(x) = \frac{x}{e^x - 1} + \frac{x}{2}$. The number $e$ is a special number that is about $2.718\ldots$. It can be found on your calculator.

6. If $f$ is an even function and $g$ is an even function...

   a) Is $f + g$ even, odd, neither, or is there just not enough information to tell? Hint: try to simplify $(f+g)(x)$ and use the assumption that both $f$ and $g$ are even.

   b) Is $f\,g$ even, odd, or neither, or is there just not enough information to tell?

7. If $f$ is an even function and $g$ is an odd function...

   a) Is $f + g$ even, odd, neither, or is there just not enough information to tell?

   b) Is $f\,g$ even, odd, or neither, or is there just not enough information to tell?

8. If $f$ is an odd function and $g$ is an odd function...

   a) Is $f + g$ even, odd, neither, or is there just not enough information to tell?

   b) Is $f\,g$ even, odd, or neither, or is there just not enough information to tell?