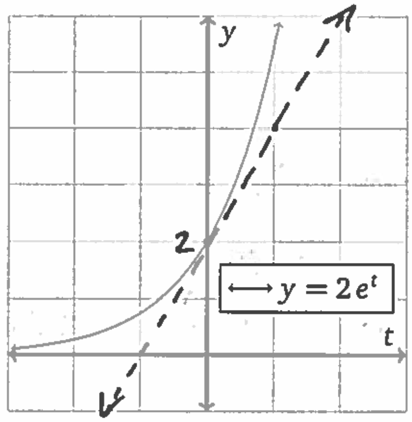
Introduction to the number e and the natural logarithm ln

In these exercises, we will familiarize ourselves more with e and what it means to have e as the growth factor in an exponential function. Also we will use the logarithm with base e, which is usually denoted ln.

1. Here is the graph of f, where $f(t) = 2e^t$.



(a) What is the initial value of this function? 2

(b) Since the function has its growth factor $b = e$, what is the initial slope? 2

(c) Sketch a straight line passing through the graph's y-intercept that has the function's initial slope. Make sure that your sketch agrees with your answers to 1a and 1b.

2. How many digits of e have you memorized? Spend a minute studying them. You can see them if you use your calculator to show you e (you may need to enter e^1). Later in this worksheet you will be asked to see how many you have memorized.

2.71828...

3. The heat energy in a chemical reaction is growing exponentially: $E(t) = a\,b^t$. Initially, there is 2300 J of heat energy. Also, for the first second of the reaction, the heat energy is growing by 2300 $\frac{\mathrm{J}}{\mathrm{h}}$.

(a) Give a formula for $E(t)$, using the observation that the initial energy *amount* is equal to the initial *rate of change* (per hour).

$E(t) = 2300 \cdot e^t$ know this is the base since initial value equals initial rate of change.

(b) Use your answer from 3a to give the growth factor per unit time (as an exact value) and the relative growth rate per unit time (as a rounded percentage).

$b = e = 2.71828\ldots$ $\quad r = b - 1 \approx 171.828\%\ldots$

(c) How long will it take until there is 5000 J of heat energy?

$5000 = 2300 \cdot e^t$

$\frac{5000}{2300} = e^t$

$\ln\left(\frac{50}{23}\right) = t$

$\Longrightarrow t \approx 0.7765\ldots$

It will take about 0.7765 hr.

4. Simplify/expand each of the following expressions, using the rules of logarithms and the rules of exponents. Do not use a calculator.

(a) $\ln(e^5)$

5

(b) $e^{\frac{1}{2}\ln 25}$

$= e^{\ln(25^{1/2})}$

$= e^{\ln(5)} = 5$

(c) $\ln\left(\frac{10}{e}\right) + \ln\left(\frac{e^2}{10}\right)$

$= \ln\left(\frac{10}{e} \cdot \frac{e^2}{10}\right) = \ln(e) = 1$

5. Solve the equations using ln or *e* somehow. Find *exact* answers without the help of a calculator. Then *also* use your calculator to find decimal approximations. Get practice using your calculator's ln command rather than any solving or graphing tool. Always check your solutions, since sometimes these equations can lead to extraneous solutions.

(a) $12^t = 40$

$\ln(12^t) = \ln(40)$

$t \cdot \ln(12) = \ln(40)$

$t = \frac{\ln(40)}{\ln(12)} \approx 1.4845...$

(c) $\ln(x-3) = \frac{1}{3}$

$e^{\ln(x-3)} = e^{1/3}$

$x - 3 = e^{1/3}$

$x = 3 + e^{1/3} \approx 4.3956$

(b) $5(3)^{2x-7} = 12\left(\frac{1}{2}\right)^{x+2}$

$\ln\left(5(3)^{2x-7}\right) = \ln\left(12\left(\frac{1}{2}\right)^{x+2}\right)$

$\ln(5) + \ln(3^{2x-7}) = \ln(12) + \ln\left(\left(\frac{1}{2}\right)^{x+2}\right)$

$\ln(5) + (2x-7)\ln(3) = \ln(12) + (x+2)\ln(1/2)$

$\ln(5) + 2x\ln(3) - 7\ln(3) = \ln(12) + x\ln(1/2) + 2\ln(\frac{1}{2})$

$2x\ln(3) - x\ln(1/2) = \ln(12) + 2\ln(1/2) - \ln(5) + 7\ln(3)$

$x[2\ln(3) - \ln(1/2)] = \ln(12) + 2\ln(1/2) - \ln(5) + 7\ln(3) \implies x = \frac{\ln(12) + 2\ln(\frac{1}{2}) - \ln(5) + 7\ln(3)}{2\ln(3) - \ln(1/2)}$

$= \frac{\ln(6561/5)}{\ln(18)} \approx 2.483$

(d) $\ln(x+3) + \ln(x-3) = 2\ln(3x-11)$

$\ln((x+3)(x-3)) = \ln((3x-11)^2)$

$\implies x^2 - 9 = (3x-11)^2 = 9x^2 - 66x + 121$

$\implies 0 = 8x^2 - 66x + 130$

$\implies 0 = 2(4x-13)(x-5) \implies x = \frac{13}{4}$ or $x = 5$

But only $x=5$ works.

6. Without using any reference, how many digits of *e* do you remember?

$2.71828...$

7. Pick a large value for n—like one million or more. Have your calculator compute $\left(1 + \frac{1}{n}\right)^n$. What is the result?

$\left(1 + \frac{1}{1000000}\right)^{1000000} = 2.71828047...$

8. Remember what $n!$ means? As an example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Have your calculator compute $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{9!}$. (You can find a ! button on your calculator in the Math menu if it is a TI calculator.) What is the result?

$\frac{1}{0!} + \frac{1}{1!} + \cdots + \frac{1}{9!} = 2.71828153...$