Symmetric Functions

In these exercises, we will use the formula and/or a table of a function to determine if the function is even, odd, or neither.

1. Determine if each of the following functions are even, odd, or neither, *by examining a graph* using your graphing calculator.

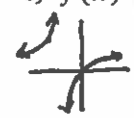
a) $f(x) = x^3 + 6x$

odd

b) $f(x) = |3x| - 7$

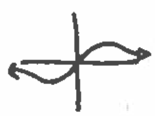
even

c) $f(x) = \dfrac{x}{x+1}$



neither

d) $f(x) = \dfrac{x}{x^2+1}$

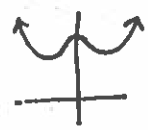


odd

e) $f(x) = 2^x$

neither

f) $f(x) = x^4 - 3x^2 + 13$



even

2. Determine if each of the following functions are even, odd, or neither, *by checking and simplifying the formula for $f(-x)$*. Remember that if $f(-x) = f(x)$, it means that you have an even function, and if $f(-x) = -f(x)$, it means that you have an odd function.

a) $f(x) = x^4$

$f(-x) = (-x)^4$
$= x^4$
$= f(x)$

f is even

b) $f(x) = |3 - x|$

$f(-x) = |3-(-x)|$
$= |3+x|$

Does $|3+x| = |3-x|$?
Does $|3+x| = -|3-x|$?
No. Just try at $x=3$.
f is neither

c) $f(x) = \dfrac{x^2}{x^2+3x+1}$

$f(-x) = \dfrac{(-x)^2}{(-x)^2+3(-x)+1}$

$= \dfrac{x^2}{x^2-3x+1}$

Does this equal $f(x)$?
Does this equal $-f(x)$?
No. Just try $x=3$
f is neither.

d) $f(x) = x^3 \cdot \sqrt{1+x^2}$

$f(-x) = (-x)^3\sqrt{1+(-x)^2}$
$= -x^3\sqrt{1+x^2}$
$= -f(x)$

f is odd

e) $f(x) = 2^x$

$f(-x) = 2^{-x} = \dfrac{1}{2^x}$

Does $\dfrac{1}{2^x} = 2^x$?

Does $\dfrac{1}{2^x} = -2^x$?

No. Just try $x=1$. f is neither.

f) $f(x) = \dfrac{x}{x-1}$

$f(-x) = \dfrac{-x}{-x-1} = \dfrac{-x}{-(x+1)}$

$= \dfrac{x}{x+1}$

Does this equal $\dfrac{x}{x-1}$?
Does this equal $-\dfrac{x}{x-1}$?
No. Just try $x=3$.
f is neither.

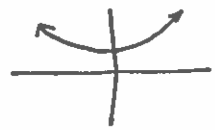
3. Why are the types of symmetry that we are studying called *even* and *odd*? What is the connection to even and odd numbers? Hint: consider a special kind of function called a *power* function. These have formulas of the form $f(x) = x^n$. Take a look at graphs of $y = x^1, y = x^2, y = x^3, \ldots$.

When $f(x) = x^n$, if n is even (or odd) as a number, then f is even (or odd) as a function.

4. There is (only) one function that is *both* even and odd. What is its formula?

$f(x) = 0$

5. It's not always clear from a formula whether a function is even or odd. Investigate the function f defined by $f(x) = \frac{x}{e^x - 1} + \frac{x}{2}$. The number e is a special number that is about 2.718.... It can be found on your calculator.

f is even. Its graph is like 

6. If f is an even function and g is an even function...

a) Is $f + g$ even, odd, neither, or is there just not enough information to tell? Hint: try to simplify $(f+g)(x)$ and use the assumption that both f and g are even.

$$\begin{aligned}(f+g)(-x) &= f(-x) + g(-x)\\ &= f(x) + g(x) \quad \text{(since } f, g \text{ even)}\\ &= (f+g)(x)\end{aligned}$$

So $f+g$ is even.

b) Is fg even, odd, or neither, or is there just not enough information to tell?

$$\begin{aligned}(fg)(-x) &= f(-x) \cdot g(-x)\\ &= f(x) \cdot g(x) \quad \text{(since } f, g \text{ even)}\\ &= (fg)(x)\end{aligned}$$

So fg is even.

7. If f is an even function and g is an odd function...

a) Is $f + g$ even, odd, neither, or is there just not enough information to tell?

$$\begin{aligned}(f+g)(-x) &= f(-x) + g(-x)\\ &= f(x) - g(x)\end{aligned}$$

is not necessarily the same as $(f+g)(x)$ or $-(f+g)(x)$.

~~f+g is neither~~ There is not enough information

b) Is fg even, odd, or neither, or is there just not enough information to tell?

$$\begin{aligned}(fg)(-x) &= f(-x) \cdot g(-x)\\ &= f(x) \cdot -g(x)\\ &= -f(x) \cdot g(x)\\ &= -(fg)(x)\end{aligned}$$

So fg is odd.

8. If f is an odd function and g is an odd function...

a) Is $f + g$ even, odd, neither, or is there just not enough information to tell?

$$\begin{aligned}(f+g)(-x) &= f(-x) + g(-x)\\ &= -f(x) - g(x)\\ &= -(f(x) + g(x))\\ &= -(f+g)(x)\end{aligned}$$

So $f+g$ is odd

b) Is fg even, odd, or neither, or is there just not enough information to tell?

$$\begin{aligned}(fg)(-x) &= f(-x) \cdot g(-x)\\ &= (-f(x)) \cdot (-g(x))\\ &= f(x) \cdot g(x)\\ &= (fg)(x)\end{aligned}$$

So fg is even.