

Plotting Rational Function Graphs by Hand

In these exercises, we will plot graphs of rational functions by hand, noting their y-intercept, their zeros, the local behavior of their zeros, the long term behavior of the function, the vertical asymptotes, and the local behavior near those vertical asymptotes.

1. Sketch a graph of each of the following functions. You need to find and label the y-intercept, the zeros, the slopes at the zeros (a.k.a the local behavior), the long-term behavior, the vertical asymptotes, and the local behavior near those vertical asymptotes. In each graph, you are responsible for setting the scale of the graph appropriately, using what you know about the y-intercept, the zeros, the slopes at the zero, and the long-term behavior.

$$a) f(x) = \frac{6x-42}{3x+5} = \frac{6(x-7)}{3(x+\frac{5}{3})}$$

$$y\text{-int: } \frac{-42}{5} = -8.4$$

$$\text{zero: } 7$$

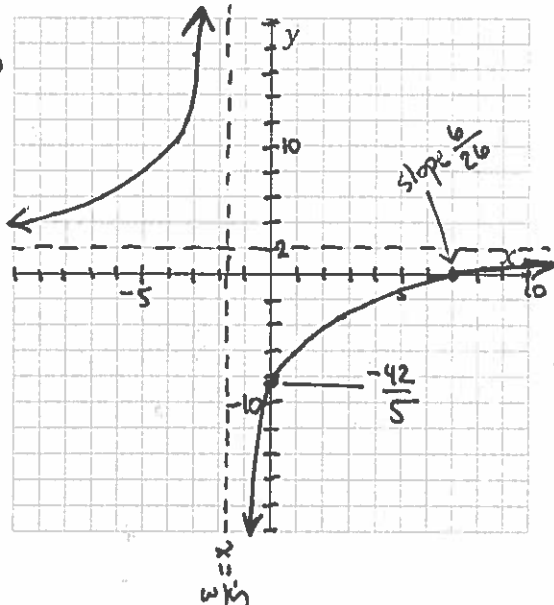
$$\begin{aligned} @ 7: & \frac{6(x-7)}{3(7+\frac{5}{3})} \\ &= \frac{6}{26}(x-7) \\ &\approx 0.23(x-7) \end{aligned}$$

vertical asymptote at $-\frac{5}{3}$

$$@ -\frac{5}{3}: \frac{6(-\frac{5}{3}-7)}{3(-\frac{5}{3}+\frac{5}{3})}$$

$$= \frac{-52}{3(x+\frac{5}{3})} = \frac{-52/3}{x+\frac{5}{3}} \approx \frac{-17.3}{x+\frac{5}{3}}$$

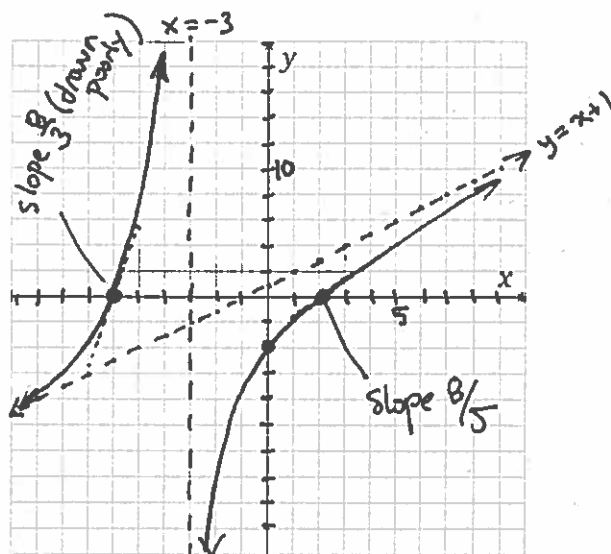
$$\text{horizontal asymptote (LTB): } y = \frac{6}{3} = 2$$



$$b) g(x) = \frac{x^2+4x-12}{x+3}$$

$$y\text{-int: } \frac{-12}{3} = -4$$

$$(0, -4)$$



$$g(x) = \frac{(x+6)(x-2)}{x+3}$$

$$\text{zeros: } -6, 2$$

$$@ -6: \frac{(x+6)(-6-2)}{(-6+3)} = \frac{-8(x+6)}{-3} = \frac{8}{3}(x+6)$$

$$@ 2: \frac{(2+6)(x-2)}{2+3} = \frac{8(x-2)}{5} = \frac{8}{5}(x-2)$$

vertical asymptote: $x = -3$

$$@ -3: \frac{(-3+6)(-3-2)}{-3+3} = \frac{-15}{x+3}$$

LTB

$$x+3 \overline{) x^2+4x-12} \rightarrow y = x+1$$

$$y\text{-int: } (0,0)$$

$$c) h(x) = \frac{x^3 + 4x^2 - 32x}{x^2 - 4x - 12} = \frac{x(x+8)(x-4)}{(x-6)(x+2)}$$

$$d) k(x) = \frac{x^2 - 6x + 9}{x^2 + 2x + 1} = \frac{(x-3)^2}{(x+1)^2}$$

$$y\text{-int: } (0,9)$$

$$@ 3: \frac{(x-3)^2}{16}$$

$$@ 0: \frac{x(3)(-4)}{(-6)(2)}$$

$$= \frac{8}{3}x$$

$$@ -8: \frac{-8(x+8)(-12)}{(-14)(-6)}$$

$$= \frac{8}{7}(x+8)$$

$$@ 4: \frac{4(12)(x-4)}{(-2)(6)}$$

$$= -4(x-4)$$

$$@ 6: \frac{6(14)(2)}{(x-6)(3)} = \frac{21}{x-6}$$

$$@ -2: \frac{(-2)(6)(-6)}{(-8)(x+2)} = \frac{-9}{x+2}$$

Using vertical ticks spaced 4 apart may be a good choice.

$$e) l(x) = \frac{(x-3)^2(x+2)}{(x+3)(x-4)^2}, \text{ which expands to } \frac{x^3 - 4x^2 - 3x + 18}{x^3 - 5x^2 - 18x + 18}$$

$$y\text{-int: } \frac{18}{19} = 1$$

$$\text{LTB: } y=1$$

$$@ 3: \frac{(x-3)^2(5)}{(6)(-1)^2}$$

$$= \frac{5}{6}(x-3)^2$$

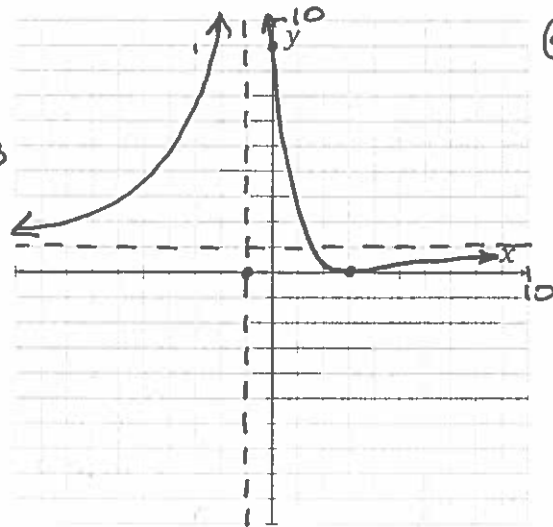
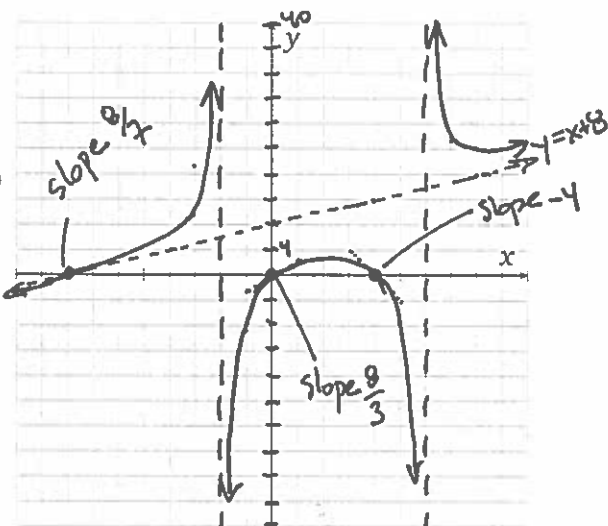
$$@ -2: \frac{(-2-3)^2(x+2)}{(-2+3)(-2-4)^2}$$

$$= \frac{25(x+2)}{36}$$

$$= \frac{25}{36}(x+2)$$

$$@ -3: \frac{(-6)^2(x-1)}{(x+3)(-7)^2}$$

$$= -\frac{36/49}{x+3}$$



$$@ -1: \frac{(-1-3)^2}{(x+1)^2}$$

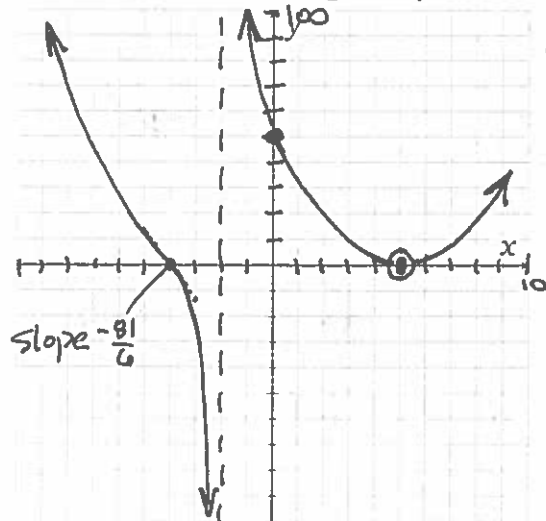
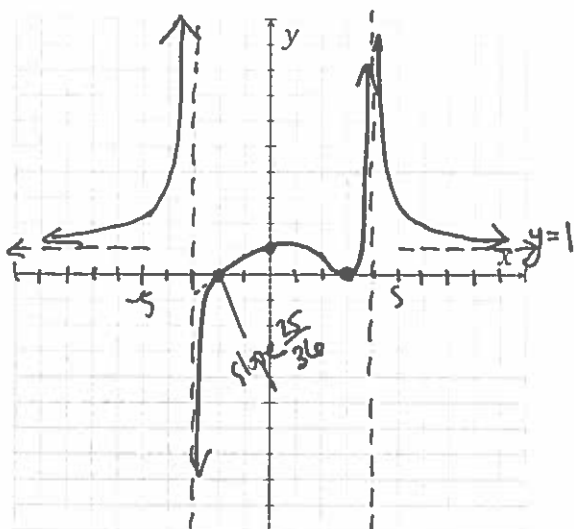
$$= \frac{16}{(x+1)^2}$$

$$y\text{-int: } \frac{4(-5)^3}{-10} = 50, (0,50)$$

$$f) m(x) = \frac{(x+4)(x-5)^3}{x^2 - 3x - 10} = \frac{(x+4)(x-5)^3}{(x-5)(x+2)} \approx \frac{(x+4)x^2}{x+2}$$

$$@ -4: \frac{(x+4)(-9)^2}{-6} = -\frac{81}{6}(x+4)$$

has LTB: $\frac{x^4 + \dots}{x^2 + \dots}$
So like x^2 .



$$@ -2: \frac{(2)(-7)^2}{x+2}$$

$$= \frac{98}{x+2}$$

Using vertical ticks spaced 10 apart may be a good choice.

$$@ 5: \frac{(9)(x-5)^2}{7} = \frac{9}{7}(x-5)^2$$

but hole at $x=5$