Polynomial Basics and Plotting Polynomial Graphs by Hand

These exercises check the basic vocabulary of polynomials. We also check that we can use a graphing calculator to learn about polynomial functions. Lastly, we plot polynomial graphs by hand, noting their y-intercept, zeros, the local behavior of their zeros, and the long term behavior of the function.

1. Identify which of these functions are polynomial functions and which are not. If the function is a polynomial function, what is its degree? What is its leading coefficient?

   a) $f(x) = 2x - 4$
   polynomial
   degree 1, leading coef 2

   b) $g(x) = 2^x + 4$
   not a polynomial

   c) $h(x) = x^2 + 18x$
   polynomial
   degree 2, leading coef 1

   d) $j(x) = x^7 + x^{3.5} - 4x$
   not a polynomial

   e) $k(x) = 3x - 14x^3 + 10$
   polynomial
   degree 3, leading coef −14

   f) $l(x) = 2x^4 - 9e^x + 2x^2$
   not a polynomial

2. Find the zeros of $x^5 - 3x^4 - 5x^2 + 4x + 2$ using your graphing calculator. Find the local minimum and local maximum using your graphing calculator. You should be using special tools like ZERO, MINIMUM, and MAXIMUM, not eyeballing a guess, not using the TRACE feature, and not using a table.
   local minimum at 2.642... with value −39.77...
   local maximum at 0.3544... with value 2.747...
   zeros at −0.3421..., 0.9161..., 3.326...

3. Find all the zeros of the given polynomial functions. You should be able to do this *without* the use of your graphing calculator.

   a) $x^2 + 8x - 20$
   $(x+10)(x-2)$
   zeros at
   -10 and 2

   b) $3x^2 - 5x - 2$
   $3x^2 - 6x + x - 2$
   $3x(x-2) + (x-2)$
   $(3x+1)(x-2)$
   zeros at
   $-\frac{1}{3}$ and 2

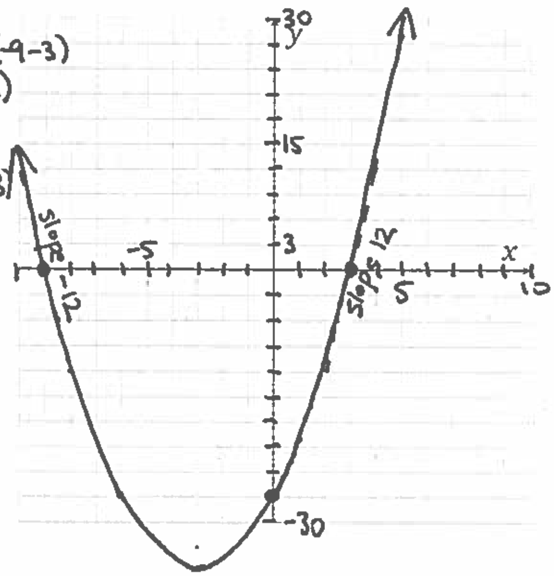
   c) $x^3 + 4x^2 + 4x$
   $x(x^2+4x+4)$
   $x(x+2)^2$
   zeros at
   0 and -2

   d) $2(3x-1)(x+3)^2$
   zeros at
   $\frac{1}{3}$ and -3

4. Sketch a graph of each of the following functions. You need to find and label the y-intercept, the zeros, the slopes at the zeros (a.ka. the local behavior), and make it clear what the long-term behavior is. In each graph, you are responsible for setting the scale of the graph appropriately, using what you know about the y-intercept, the zeros, and the slopes at the zeros.

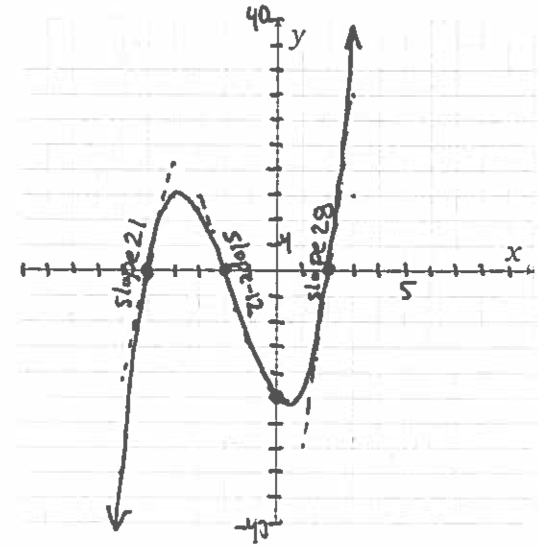
a) f, where $f(x) = x^2 + 6x - 27$
$= (x+9)(x-3)$

y-int: $(0,-27)$
zeros: -9 and 3
@-9: $f(x) \approx (x+9)(-9-3)$
$f(x) = -12(x+9)$
slope -12
@ 3: $f(x) \approx (3+9)(x-3)$
$f(x) \approx 12(x-3)$
slope 12



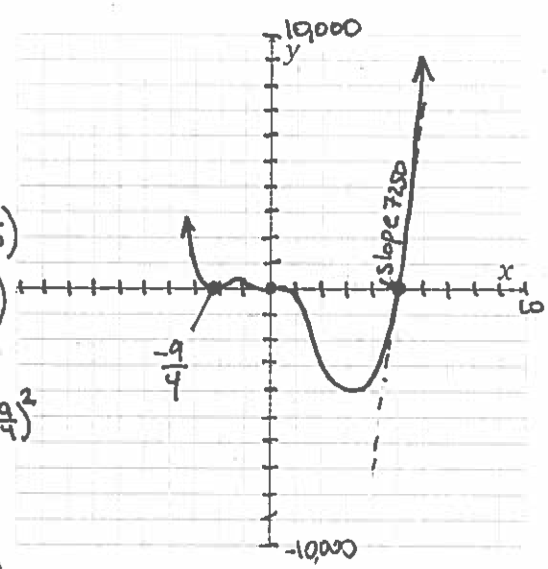
b) g, where $g(x) = (x-2)(x+2)(x+5)$
y-int: $(-2)(2)(5) = -20 \rightarrow (0,-20)$

zeros: $2, -2, -5$
@2: $(x-2)(4)(7) = 28(x-2)$
@ -2: $(-4)(x+2)(3) = -12(x+2)$
@ -5: $(-7)(-3)(x+5) = 21(x+5)$



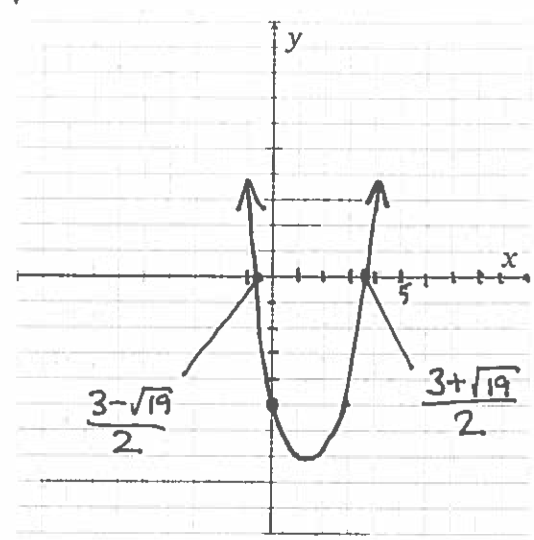
c) h, where $h(x) = x^3(4x+9)^2(x-5)$

y-int: $(0,0)$
Zeros at $0, \frac{-9}{4}, 5$
@ 0: $x^3(9)^2(-5) = -405x^3$
@ $\frac{-9}{4}$: $\left(\frac{-9}{4}\right)^3(4x+9)^2\left(\frac{-9}{4}-5\right)$
$= \frac{-729}{64}(4x+9)^2\left(\frac{-29}{4}\right)$
$= \frac{21141}{256}(4x+9)^2$
$= \frac{21141}{16}\left(x+\frac{9}{4}\right)^2$
@ 5: $5^3(29)^2(x-5)$
$= 7250(x-5)$



d) k, where $k(x) = 2x^2 - 6x - 5$
y-int: $(0,-5)$

Zeros:
$2x^2-6x-5=0$
$x = \frac{6 \pm \sqrt{36+40}}{4}$
$= \frac{6\pm\sqrt{76}}{4}$
$= \frac{3\pm\sqrt{19}}{2}$
$\approx 3.67...,$
or
$\approx -0.67..$



(You may skip the slopes for this one)