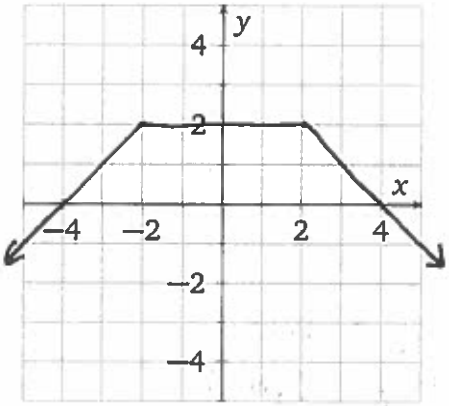
Piecewise-Defined Functions

In these exercises, we will practice plotting a piecewise-defined function. We will also practice reading the graph of a piecewise-defined function.

1. Graph the piecewise defined function by hand. If it helps, plot points first. Use arrows to indicate if a graph extends of the grid in some way. Us solid and hollow dots to indicate special behavior of the function where there is a transition from one rule to another rule.



$$f(x)=\begin{cases} x+4, & x\le -2 \\ 2, & -2<x<2 \\ 4-x, & x\ge 2 \end{cases}$$

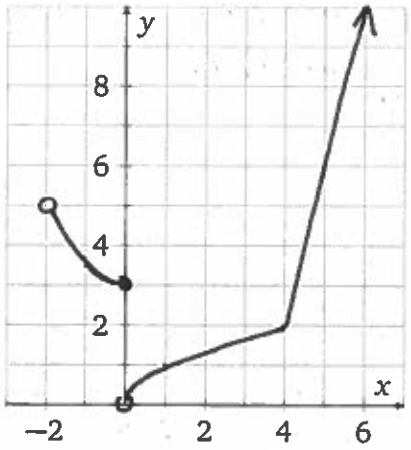
What is the domain and range of f?

Need a graph sketched first

$(-\infty, 2]$

$(-\infty,-2]\cup(-2,2)\cup[2,\infty)$ which simplifies to $(-\infty,\infty)$

2. Graph the piecewise defined function by hand. If it helps, plot points first. Use arrows to indicate if a graph extends of the grid in some way. Us solid and hollow dots to indicate special behavior of the function where there is a transition from one rule to another rule.



$$f(x)=\begin{cases} \frac{1}{2}x^2+3, & -2<x\le 0 \\ \sqrt{x}, & 0<x<4 \\ 4(x-4)+2, & x\ge 4 \end{cases}$$

What is the domain and range of f?

Make the graph first

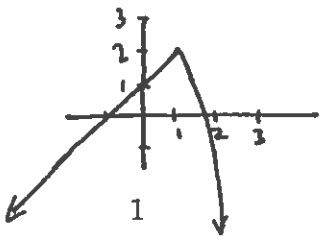
$(0,\infty)$

$(-2,0]\cup(0,4)\cup[4,\infty)$ which simplifies to $(-2,\infty)$

3. Consider the function f defined by $f(x)=\begin{cases} x+1, & x<1 \\ 3-x^2 & x\ge 1 \end{cases}$. What is the domain of this function and what is its range?

$(-\infty,1)\cup(1,\infty)$ which simplifies to $(-\infty,\infty)$

By now we know that making a graph is necessary.



range is $(-\infty, 2]$

4. Use your calculator to graph $f(x) = \begin{cases} x+1, & x<1 \\ 3-x^2 & x \geq 1 \end{cases}$.

TI-89: In the Y= menu, enter:

$Y1 = \text{when}(x<1, x+1, \text{when}(x \geq 1, 3-x^2, 1/0))$

You can either type "when" directly or use the CATALOG. Use the Math⟶Test menu to get the symbols $<$ and $\geq$.

TI-83/84: In the Y= menu, enter:

$Y1 = (x+1)*(x<1) + (3-x^2)*(x \geq 1)$

Use the Math⟶Test menu to get the symbols $<$ and $\geq$.

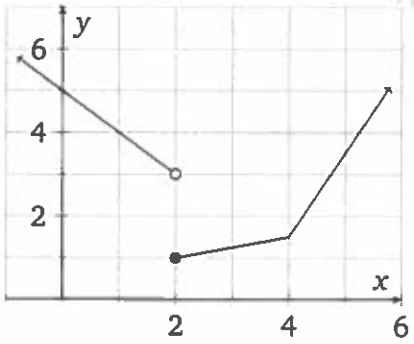
Casio 330: In the Y= screen, enter:

$Y1 = \begin{cases} x+1, & x<1 \\ 3-x^2 & x \geq 1 \end{cases}$

Use the 2D tab and the special button for piecewise-defined functions.

Go to the Graph screen, making sure that you are in a good Window.

5. Find a piecewise defined formula for the function with this graph.



left piece: $y = -x+5$

middle piece: $y = \frac{1}{4}(x-2)+1$

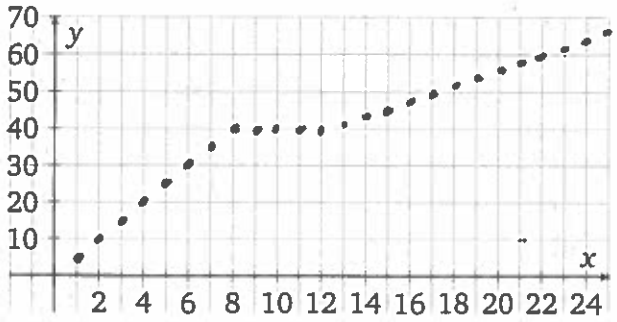
right piece: $y = 2(x-4)+1.5$

So $f(x) = \begin{cases} -x+5, & x<2 \\ \frac{1}{4}(x-2)+1, & 2 \leq x \leq 4 \\ 2(x-4)+1.5, & x>4 \end{cases}$

6. A museum charges \$5 per person. However, larger groups get a discount. You may purchase a group ticket for \$40 that lets in up to 12 people. If your group has more than 12 people, each additional person costs \$2 more.

Let n be the number of people in a group that wants to enter the museum. Let C be the entrance cost for the entire group. The cost is a function of how many people are in the group: $C = f(n)$

Plot $f(n)$ for $n = 1$ up to $n = 20$, and give a piecewise defined formula for $f(n)$.



$f(1) = 5$
$f(2) = 10$
$f(3) = 15$
⋮
$f(8) = 40$
$f(9) = 40$
$f(10) = 40$
$f(11) = 40$
$f(12) = 40$
$f(13) = 42$
$f(14) = 44$
⋮

$f(n) = \begin{cases} 5n, & 1 \leq n \leq 8 \\ 40, & 8 < n \leq 12 \\ 2(n-12)+40, & n>12 \end{cases}$