Introduction to Logarithms

In these exercises, familiarize yourself with some basic facts about logarithm functions and use them to solve some equations. Write both exact solutions as well as decimal approximations.

TI-89 to find $\log_b(a)$, type `log(a,b)`

Casio to find $\log_b(a)$, type `log(b,a)`

TI-83,84 no direct way to enter logarithms with other bases—skip the decimal approximation for now. Later we will learn an alternative.

1. Solve the equations using logarithms. Find *exact* answers (like $\log_3(7)-2$) without the help of a calculator. Then *also* use your calculator to find decimal approximations.

   (a) $12^x = 40$

   $x = \log_{12}(40)$

   $\approx 1.4845\ldots$

   (b) $2(3)^{2x+1} = 17$

   $3^{2x+1} = 17/2 = 8.5$

   $2x+1 = \log_3(8.5)$

   $2x = \log_3(8.5) - 1$

   $x = \frac{1}{2}(\log_3(8.5) - 1) \approx 0.47398\ldots$

   (c) $\dfrac{20}{5^x - 8} = 2$

   $20 = 2 \cdot 5^x - 16$

   $36 = 2 \cdot 5^x$

   $18 = 5^x$

   $x = \log_5(18)$

   $\approx 1.7958\ldots$

   (d) $17(0.18)^t = 2^t$

   $\dfrac{(0.18)^t}{2^t} = 1/17$

   $0.09^t = \frac{1}{17}$

   $t = \log_{0.09}(\frac{1}{17})$

   $\approx 1.1766\ldots$

2. If a population started at 250 million and grew exponentially with a relative growth rate of 9.2% per year, how long will it be until the population has doubled? Get practice using your calculator's `log` command rather than any solving or graphing tool.

   $250(1.092)^t = 500$

   $1.092^t = 2$

   $t = \log_{1.092}(2) \approx 7.8756$

   It would take 7.8756... years to double.

3. Evaluate the following *without* a calculator.

(a) $\log(10^4)$ $= 4$

(b) $\log(1000)$ $= 3$

(c) $\log(\frac{1}{1000})$ $= -3$

(d) $\log(0.01)$ $= -2$

(e) $\log(\sqrt{10})$ $= \frac{1}{2}$

(f) $\log(\sqrt[3]{100})$ $= 2/3$

4. A bank account grows exponentially and has a relative growth rate of 3.2% per year. If we invest \$1000, how long will it be until we have earned \$200 in interest? Get practice using your calculator's `log` command rather than any solving or graphing tool.

$$1000(1.032)^t = 1200$$
$$1.032^t = 1.2$$
$$t = \log_{1.032}(1.2)$$
$$\approx 5.78823\ldots$$

It would take about 5.78823 yrs to earn \$200 in interest.

5. A radioactive substance decays exponentially and has a relative decay rate of 14%/month. If we start with 1000 mg, how long will it be until we only have one mg left? Get practice using your calculator's `log` command rather than any solving or graphing tool.

$$1000(0.86)^t = 1$$
$$0.86^t = 0.001$$
$$t = \log_{0.86}(0.001)$$
$$\approx 45.8$$

It would take about 45.8 months to decay from 1000 mg to only 1 mg.