Properties of Logarithmic Functions

In these exercises, we will see work with the domain and range of logarithmic functions. And we will use graph transformations to sketch graphs.

1. Find the domain ~~and range~~ of each of the following functions.

a) f, where $f(x) = \ln(x-4)$

Need $x-4>0$
$\Rightarrow x>4$
$(4,\infty)$

b) g, where $g(x) = \log_2(2x+14)$

Need $2x+14>0$
$\rightarrow 2x>-14$
$\Rightarrow x>-7$
$(-7,\infty)$

c) b, where $b(x) = \ln(10-x)$

Need $10-x>0$
$-x>-10$
$x<10$
$(-\infty, 10)$

d) c, where $c(x) = \log_7(14-2x)$

Need $14-2x>0$
$-2x>-14$
$x<7$
$(-\infty, 7)$

e) p, where $f(x) = \ln(x-4)+\ln(10-x)$

Need $x-4>0$ AND $10-x>0$
$x>4$ $-x>-10$
$x<10$

$\Rightarrow$ domain is $(4,10)$

f) q, where $q(x) = \frac{\log(x-6)}{\log(x+1)}$

Need $x-6>0 \Rightarrow x>6$
AND $x+1>0 \Rightarrow x>-1$
AND $\log(x+1)\neq 0 \Rightarrow x+1\neq 10^0$
$x+1\neq 1$
$x\neq 0$

To have $x>6$, $x>-1$, and $x\neq 0$
$\Rightarrow$ domain is $(6,\infty)$

g) h, where $h(x) = \log\left(\left|\frac{x+1}{x-6}\right|\right)$

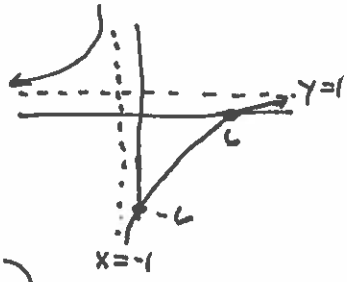
At $x=6$, the expression is undefined (division by 0). At $x=-1$, the expression is undefined ($\ln(0)$ is not defined). For all other x, $\left|\frac{x+1}{x-6}\right|$ is positive. So the domain is $(-\infty,-1)\cup(-1,6)\cup(6,\infty)$

h) k, where $k(x) = \log\left(\frac{x-6}{x+1}\right)$

Need $\frac{x-6}{x+1}>0$

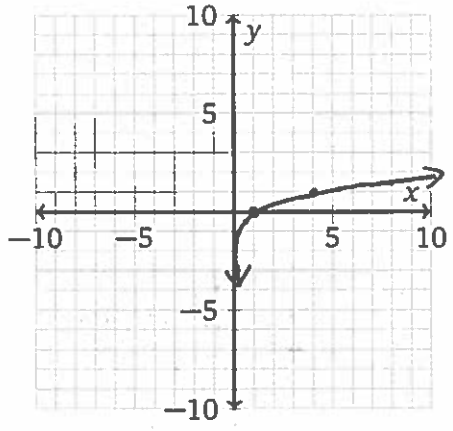
$\Downarrow$

domain is $(-\infty,-1)\cup(6,\infty)$

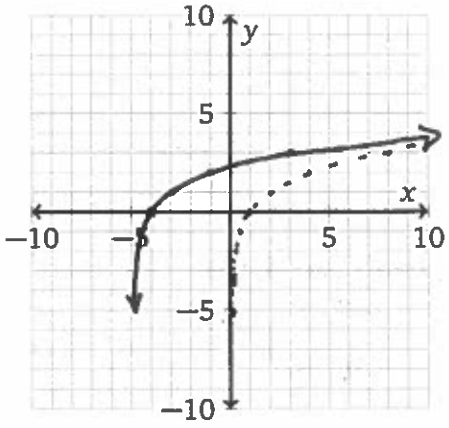


2. Sketch a graph for each of the following functions without using a graphing calculator. It very much helps to remember that $\log_b(\ )$ is the inverse function of $b^{(\ )}$.

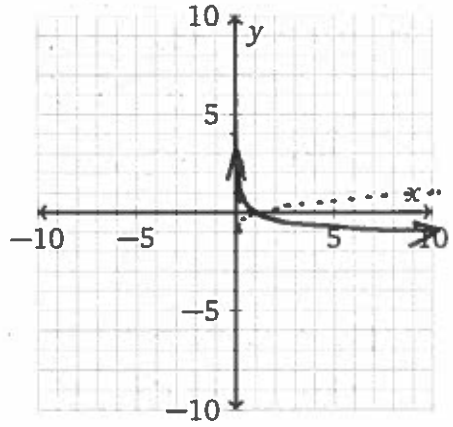
a) f, where $f(x) = \log_4(x)$



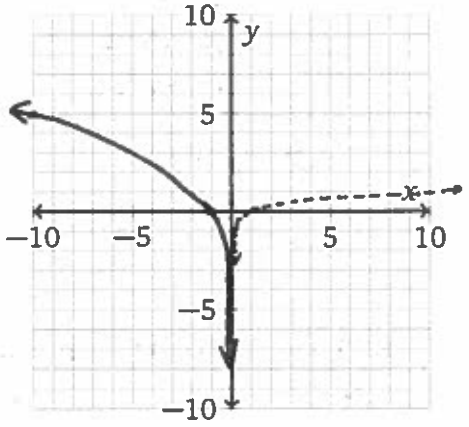
b) g, where $g(x) = \log_2(x+5)$



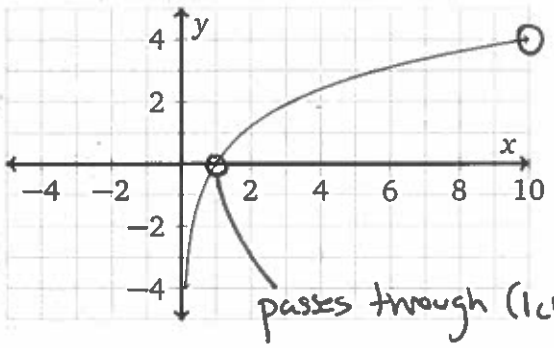
c) h, where $h(x) = -\log(x)$



d) h, where $h(x) = 5\log(-x)$



3. Find a formula for the function plotted. Hints: what basic function shape does this graph resemble? Can you identify some specific grid points that it passes through?



passes through (10,4)

could be log(x) (which passes through (10,1)) stretched vertically by 4.

passes through (1,0) just like $\log_b(x)$

$f(x) = 4 \cdot \log(x)$