

Inverses of Functions

In these exercises, we will take a given (invertible) function and represent its inverse function with a formula, with a table, verbally, and graphically.

1. Find formulas for the inverse functions of these functions. As a reminder, formulas for functions can use anything for the dummy variable. For instance, the formulas $q(x) = x^2$, $q(y) = y^2$, and $q(A) = A^2$ all say the same thing: q is a function that squares its input. This might clear up some confusion about how to write formulas for the inverse functions below.

a) $y = f(x)$, where $f(x) = 2x - 9$

f doubles, subtracts 9
 f^{-1} adds 9, cuts in half.
 f turns x into y
 f^{-1} turns y into x

$$f^{-1}(y) = \frac{y+9}{2}$$

b) $P = g(t)$, where $g(t) = \sqrt[3]{30t} + 1$

g mult. by 30, cube roots, adds 1
 g^{-1} subtracts 1, cubes, divides by 30
 g turns t into P
 g^{-1} turns P into t

$$\Rightarrow g^{-1}(P) = \frac{(P-1)^3}{30}$$

c) $C = h(w)$, where $h(w) = \frac{w}{3+2w}$

$h(w) = \frac{w}{3+2w}$
 $C = \frac{w}{3+2w}$
 $C \cdot (3+2w) = w$
 $3C + 2Cw = w$
 $3C = w - 2Cw$

$$3C = w(1-2C)$$

$$\frac{3C}{1-2C} = w$$

$$\Rightarrow h^{-1}(C) = \frac{3C}{1-2C}$$

2. Use the table to make a table of values for f^{-1} .

x	2	4	6	8	10
$f(x)$	8	2	4	1	10

x	1	2	4	8	10
$f^{-1}(x)$	8	4	6	2	10

3. The perimeter of a square of side s (measured in cm) is given by $P = f(s)$, where $f(s) = 4s$, and P is measured in cm. Find the following, and then interpret what you have found (using everyday-language):

a) $f(3) = 4 \cdot 3$
 $= 12$

A square of side length 3cm has perimeter 12cm

b) $f^{-1}(20) = \frac{20}{4}$
 $= 5$

A square of perimeter 20cm has side length 5cm

c) $f^{-1}(P) = \frac{P}{4}$

A square of perimeter P has side length $\frac{P}{4}$.

4. The gross domestic product (GDP) of the US is given by $G(t)$, where t is the number of years since 1990 and the units of G are billions of dollars.

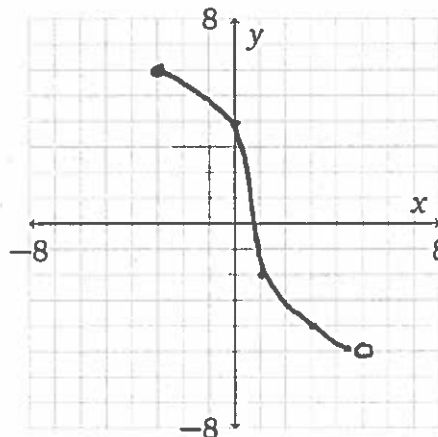
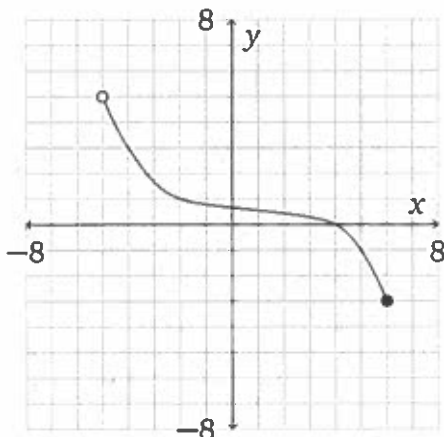
a) What does $G(11)$ represent?

The GDP in 2001. (in billions of dollars)

b) What does $G^{-1}(9873)$ represent?

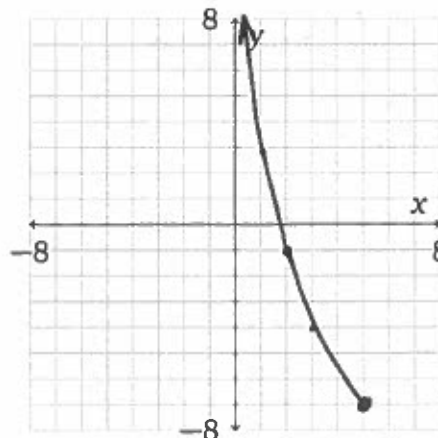
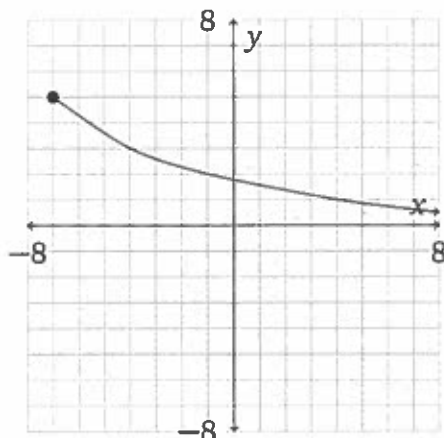
The number of years between 1990 and the year when the GDP was \$9873 billion.

5. Let $y = f(x)$, where f is defined by the graph below.



Graph the curve $y = f^{-1}(x)$. Use the specific input-output pairs that you found in part (a), but plot them appropriately for a graph of f^{-1} .

6. Plot a graph of g^{-1} if g is defined by the graph below.



7. (Challenging) Use your calculator to graph both

$$f(x) = \frac{1}{2}x^3 + \frac{3}{2^{1/3}}x \text{ and } g(x) = (\sqrt{x^2 + 4} + x)^{1/3} - (\sqrt{x^2 + 4} - x)^{1/3}.$$

Then show that f and g are inverse functions of each other. To do this, you will need a lot of space on separate paper to correctly simplify $f(g(x))$ and $g(f(x))$. This exercise is a challenge to you that you can succeed with if you carefully apply basic algebra. It is too challenging for an exam though.