# Introduction to Functions

In these exercises, we work with basic vocabulary and ideas associated with functions and relations.

1. Writing function formulas:

   a) Let $f$ be a function that triples its input, and then subtracts 4. Write a formula for this function.

   $f(x) = 3 \cdot x - 4$

   b) Let $b$ be a function that squares its input, then divides by two, and then adds 1. Write a formula for this function.

   $b(x) = \frac{x^2}{2} + 1$

   c) Barack Obama was born on August 4th, 1961. Let "age" be a function that finds his age on August 5th in year $x$. Write a formula for this function. Hint, you may want to calculate his age in years like 2000, 2008, and 2016 first to get a feel for what the formula will look like.

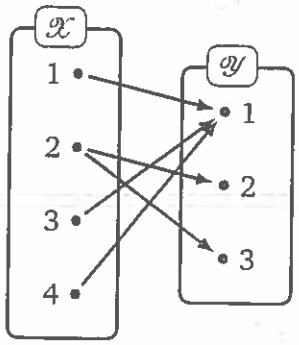
   $age(x) = x - 1961$

   d) Suppose that income tax is collected at a flat rate of 15%. Let "tax" be the function that finds what your income tax amount is for the year based on what your annual income is. Write a formula for this function.

   $tax(x) = 0.15x$

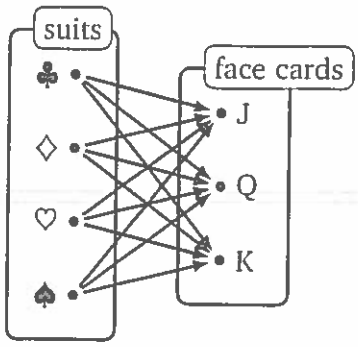
2. Below are several diagrams of *relations*. Which of these relations can be thought of as *functions*? For each, give an explanation why or why not.

a)



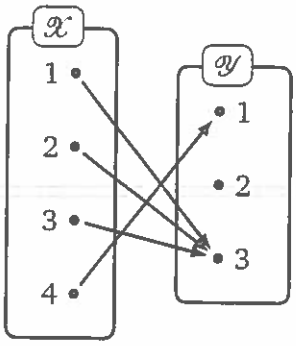

No, what would f(2) be?

b)




No. Each input has multiple outputs.

c)




Yes. f(1), f(2), f(3), and f(4) are all clearly defined.

3. For each of these relations between $x$ and $y$, decide if $y$ is a function of $x$. For each relation, after you make your decision, say a little bit about why. If you decide that the relation *can* be used to define a function, then you can just say that. But if it *cannot*, then give a reason why. For example, you might say something like "2 is in the domain, but it is unclear what $f(2)$ would be."

a) $y = 3x - 2$

Yes

b) $2y = 3x - 2$

Yes

c) $y^2 = 3x - 2$

No. For example if $x=1$, $y$ could be 1 or $-1$.

d) $x^2 + y^2 = 4$

No. For example if $x=0$, $y$ could be 2 or $-2$.

e) $\dfrac{1}{x+y} = 13$

Yes

f) $|y| - x^3 = x^2$

No. For example if $x=1$, $y$ could be 2 or $-2$.

g) $y = \pm\sqrt{x}$

NO. This equation explicitly tells you that x-values lead to two y-values.

h) $x = \dfrac{y+1}{y-2}$

Yes:

$x(y-2) = y+1$

$xy - 2x = y+1$

$xy - y = 2x+1$

$y(x-1) = 2x+1$

$y = \dfrac{2x+1}{x-1}$

4. Verbal interpretation: in the following scenarios, is $y$ a function of $x$? If not, explain why not.

a) Let $x$ represent the height of a person (a person in general, not one specific person), and $y$ represent weight.

No. The same height can be associated with more than one weight.

b) Let $x$ represent the square footage of the walls in a regular rectangular room that you need to wallpaper, and $y$ represent the number of rolls of wallpaper that you will need. (Your wallpaper store sells it in rolls of 100 square feet.)

c) Let $x$ represent the number of beers that Julia has drank in the past hour, and $y$ represent her blood alcohol content.

Depends on your interpretation of the question. Do single values for $x$ allow for being paired with more than one $y$-value?

5. Let

$$f(x) = x^2 - 4x, \quad g(x) = \frac{\sqrt{x-4}}{x^{4/3}}$$

and $h(x)$ be the number of eggs you have if you bought $x$ dozen eggs. Evaluate each of the following.

a) $f(10)$

$= 100 - 40$

$= 60$

b) $f(-3)$

$= 9 + 12$

$= 21$

c) $g(4)$

$= \dfrac{\sqrt{4-4}}{4^{4/3}} = 0$

d) $f(8)$ $= 64 - 32$

$= 32$

e) $h(3)$

$= 36$

(since 3 dozen eggs is 36 eggs)

f) $f(5) + h(9)$

$= 25 - 20 + 108$

$= 113$

g) $2h(3) - h(1)$

$= 2 \cdot 36 - 12$

$= 60$

h) $\dfrac{g(125)}{f(1) + h(2)}$

$= \dfrac{\dfrac{\sqrt{125-4}}{125^{4/3}}}{1 - 4 + 24}$

$= \dfrac{\frac{11}{625}}{21} = \dfrac{11}{13125}$