

## Introduction to Domain and Range

In these exercises, we practice identifying domains of functions based on a formula, a verbal description, a relational presentation, or a diagram. In some cases, we also identify the function's range.

1. Find the domains of the functions described here. Express each domain in *interval notation*.

a)  $q$ , where  $q(x) = x^2 + x + 1$

$$(-\infty, \infty)$$

b)  $f$ , where  $f(x) = \frac{3}{x-5}$

$$(-\infty, 5) \cup (5, \infty)$$

c)  $g$ , where  $g(x) = \frac{24}{x^2 + 1}$

$$(-\infty, \infty)$$

d)  $h$ , where  $h(x) = \frac{x^2 - 3x + 2}{x^2 + x - 20}$

$$(-\infty, -5) \cup (-5, 4) \cup (4, \infty)$$

e)  $\text{force}$ , where  $\text{force}(y) = \frac{2}{y(y-8)}$

$$(-\infty, 0) \cup (0, 8) \cup (8, \infty)$$

f)  $k$ , where  $k(t) = \frac{|t|}{t}$

$$(-\infty, 0) \cup (0, \infty)$$

g)  $\ell$ , where  $\ell(x) = \sqrt{x+43}$

$$[-43, \infty)$$

h)  $\text{squee}$ , where  $\text{squee}(x) = \sqrt{14-9x}$

$$(-\infty, \frac{14}{9}]$$

i)  $p$ , where  $p(x) = \frac{2}{|x|-8}$

$$(-\infty, -8) \cup (-8, 8) \cup (8, \infty)$$

j)  $c$ , where  $c(a) = \sqrt[3]{a^3 - 3a^2 + 23}$

$$(-\infty, \infty)$$

2. Find the domain of these functions which are described verbally. Express the domain however you see fit. In some cases, it is reasonable to have answers that differ from what your neighbor thinks, and yet you are both correct.

a)  $\text{age}(t)$  is the age of Queen Elizabeth II on April 22 of year  $t$ . Queen Elizabeth II was born on April 21, 1926.

$$\{1926, 1927, 1928, \dots\}$$

or

$$[1926, \infty)$$

b)  $\text{revenue}(n)$  is the revenue at a movie theater when they sell  $n$  tickets to a particular show. This theater has 300 seats.

$$\{0, 1, 2, \dots, 300\}$$

or

$$[0, 300]$$

c)  $q(x)$  is the number of points that the home team has  $x$  minutes after an NBA game begins. An NBA basketball game lasts 48 minutes.

$$[0, 48]$$

3. What is the *range* of these functions? To do this, you really have to think about what kinds of numbers are possible outputs. For now, there is no other way.

a)  $f(x) = x + 17.$

$$(-\infty, \infty)$$

b)  $f(x) = x^2.$

$$[0, \infty)$$

c)  $f(x) = \frac{1}{x}.$

$$(-\infty, 0) \cup (0, \infty)$$

4. A function named  $d$  is given by  $\{(1, 2), (2, 4), (3, 8), (4, 8), (5, 3)\}$ .

a) What are the domain and range of  $d$ ? Give your answers using set notation.

$$\text{domain} = \{1, 2, 3, 4, 5\}$$

$$\text{range} = \{2, 3, 4, 8\}$$

b) Evaluate  $d(3).$  = 8

c) Solve  $d(x) = 3.$

$$x = 5$$

5. In order to make table sugar, a factory had to purchase \$20,000 of machinery. Beyond that, there are costs of materials, labor, and transportation that go into producing  $x$  pounds of sugar. If  $f(x)$  is the total cost in dollars to a factory that will make  $x$  pounds of sugar, what are the domain and range for  $f$ ?

$$\text{domain of } f \text{ is } [0, \infty)$$

$$\text{range of } f \text{ is } [20000, \infty)$$