Fitting an Exponential Formula to Two Points

In these exercises, we will practice taking information about two points on the plane and finding the one and only exponential function whose graph passes through those points. We will also see an application of this skill.

go back 4 days

1. Find an exponential function f satisfying the given conditions.

(a) $f(0) = 18$ and $f(3) = 2.25$

Since $f(0)=18$, $a=18$.

$f(t)=18\cdot b^t$

$2.25 = 18\cdot b^3$

$\frac{2.25}{18} = b^3$

$\frac{1}{8} = b^3$

$\frac{1}{2} = b$

$\Longrightarrow f(t) = 18\left(\frac{1}{2}\right)^3$

(c) $f(4) = 20$ and $f(11) = 12$

starting output

forward distance from 4 to 11

ending output

$20\cdot b^7 = 12$

$\Longrightarrow b^7 = \frac{3}{5}$

$\Longrightarrow b = \sqrt[7]{3/5}$

$a = 20 \div \left(\sqrt[7]{3/5}\right)^4$

$= 20 \div (3/5)^{4/7}$

$= 20\left(\frac{5}{3}\right)^{4/7}$

So $f(t) = 20\left(\frac{5}{3}\right)^{4/7}\cdot\left(\sqrt[7]{3/5}\right)^t \approx 26.78\,(0.9296\ldots)^t$

(b) $f(3) = 6$ and $f(9) = 12$

$6\cdot b^6 = 12$

$b^6 = 2$

$b = \sqrt[6]{2}$

$a = 6 \div \left(\sqrt[6]{2}\right)^3$

$a = 6 \div (\sqrt{2}) = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

$f(t) = 3\sqrt{2}\left(\sqrt[6]{2}\right)^t$

$\approx 4.243\,(1.12246\ldots)^t$

(d) $f(-8) = \frac{1}{2}$ and $f(2) \approx 1.234\ldots$

$\frac{1}{2}\cdot b^{10} = 1.234\ldots$

$b^{10} = 2.468\ldots$

$b \approx \sqrt[10]{2.468\ldots} = 1.0945\ldots$

$a = \frac{1}{2}(1.0945\ldots)^8 = 1.03002\ldots$

$\Longrightarrow f(t) \approx 1.03002\ldots(1.0945\ldots)^t$

2. A curve passes through the points $(12, 4)$ and $(3, 6)$.

(a) If the curve is the graph of an exponential function, find a formula for that function. What is its relative growth rate per unit x? What is its growth factor per unit x?

$12-3$

$6\cdot b^9 = 4$

$b^9 = 2/3$

$b = \sqrt[9]{2/3} = 0.9559\ldots$

$a = 6 \div b^3$

$= 6 \div (0.9559\ldots)^3$

$= 6.868\ldots$

$\Longrightarrow f(t) = 6.868\ldots(0.9559\ldots)^t$

$b \approx 0.9559\ldots$

$r = b - 1$

$\approx -4.41\%$

(b) If the curve is the graph of a *linear* function, find a formula for that function. What is its slope?

$m = \frac{6-4}{3-12} = \frac{2}{-9}$ (slope $-\frac{2}{9}$)

$f(x) = -\frac{2}{9}(x-3)+6$

3. The population of the United States has increased roughly exponentially from 76.2 million people in 1900 to 309 million people in 2010.[1]

   (a) Find a formula that approximates the number of people in the U.S.A., $p(t)$, in millions, t years after 1900 assuming that the population grows exponentially.

   $t=0 \Rightarrow a = 76.2$. $\quad 76.2 \cdot b^{110} = 309 \Rightarrow b = \left(\frac{309}{76.2}\right)^{1/110} \approx 1.012808\ldots$

   $\Longrightarrow P(t) = 76.2\,(1.012808\ldots)^{t}$

   (b) Use your model to approximate the population of the U.S.A. in the year 2000.

   $P(100) \approx 76.2\,(1.012808\ldots)^{100} \approx 272$ (million)

   (c) The actual population of the U.S.A. in 2000 was about 282 million people.[2] Did your approximation from 3b underestimate or overestimate, and by how much?

   It underestimated by 10 million.

   reset so this is $t=0$ $\rightarrow a = 282$

   (d) Use the actual populations of the U.S.A. in the years 2000 and 2010 to find a formula that approximates the number of people in the U.S.A., $P(t)$, in millions, t years after 2000 assuming that the population grows exponentially.

   $282 \cdot b^{10} = 309$

   $b = (309/282)^{1/10}$

   $b \approx 1.00918535\ldots$

   $P(t) = 282\,(1.00918535\ldots)^{t}$

   Should say 3a

   (e) The population of the U.S.A. was 151 million people in 1950[3]. How close of an approximation does your model from 3d give? How close of an approximation does your model from 3 give? Why are both models off? Which model is better for making a prediction about 1950?

   Should say 3d.

   3a $\Rightarrow P(50) = 76.2\,(1.012808\ldots)^{50} \approx 144$

   3d $\Rightarrow P(-50) = 282\,(1.00918535)^{-5} \approx 178.5$

   this model is closer (the one that used 1900 & 2010)

[1]http://www.wolframalpha.com/input/?i=us+population+1900
[2]http://www.wolframalpha.com/input/?i=us+population+2000
[3]http://www.wolframalpha.com/input/?i=us+population+1950