Introduction to Exponential Functions

In these exercises, we will practice with the basic vocabulary, formulas, and graph shapes for exponential functions.

1. Write a formula for an exponential function to model these situations.

a) You have an overdue library book. The initial fine was \$0.50, but that fine collect 1% interest each week. What are the relative growth rate and the growth factor per week? Write a formula for the the function that computes your debt to the library t weeks after the due date.

initial value $a = 0.50$

1% is added each week

$\Rightarrow r = 0.01/\text{week}$

$\Rightarrow b = 1 + r$

$b = 1.01$

$A(t) = 0.50(1.01)^t$

b) A bucket of 8 nonnative fish was dumped into a lake, and the population of these fish begins to double each month. What are the relative growth rate and the growth factor per month? Write a formula for the the function that computes the population t months after the dumping.

initial value $a = 8$

Multiply by 2 each month $\Rightarrow b = 2/\text{month}$

$\Rightarrow r = b - 1 = 2 - 1 = 1 = 100\%/\text{month}.$

$P(t) = 8 \cdot 2^t$

c) The rain forest square mileage is shrinking by 15% per decade. What is the relative growth rate and the growth factor per decade? Write a formula that computes the square mileage of rain forest t decades from now, assuming that right now the square footage is A_0.

Each decade Subtract 15%

$\Rightarrow r = -0.15/\text{decade}$

$\Rightarrow b = 1 + r$

$= 0.85$

initial value

$A(t) = A_0(0.85)^t$

d) A stock initially valued at \$50 increases by 10% each month for 10 months, and then decreases by 10% each month for 10 months. How much is it worth now? (Hint: first consider an exponential growth function, see where you are 10 months later, and then reset the initial value and consider an exponential *decay* function.

initial

$50(1.10)^{10}(0.90)^{10} = 45.22$

increase by 10%   for 10 months   decrease by 10%   for 10 months

You'll only end up with \$45.22

e) Assume that inflation makes prices rise by 3.5% per year. If a dozen eggs cost \$2 in 2000, what will a dozen eggs cost in 2020? (Assume that inflation is the only factor influencing price change.) What is the one-decade inflation rate?

$r = 0.035/\text{year}$

$\Rightarrow b = 1.035$

$\Rightarrow A(t) = 2(1.035)^t$

$A(20) = 2(1.035)^{20}$

$= 3.98$

In 2020, a dozen eggs will cost about \$3.99

$A(10) = 2(1.035)^{10}$

$= 2(1.41\ldots)$

So the one-decade inflation rate is about 41%

2. Without using a graphing calculator, match each exponential function with its graph. Use what you can see in the formulas about growth factors and initial values.

(a) $5(1.3)^t$ = $q(t)$

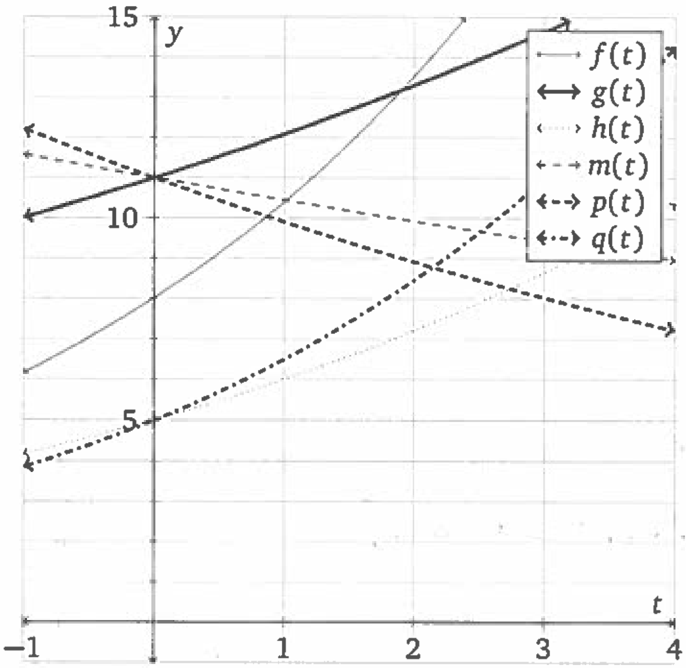
(b) $5(1.2)^t$ = $h(t)$

(c) $8(1.3)^t$ = $f(t)$

(d) $11(0.9)^t$ = $p(t)$

(e) $11(0.95)^t$ = $m(t)$

(f) $11(1.1)^t$ = $g(t)$



3. Here are some exponential functions. Identify the initial value, the growth factor per unit t, and the relative growth rate per unit t. In the case of the relative growth rate, express it as a percentage.

a) $Q(t) = 50.3(1.183)^t$

$a = 50.3$ $b = 1.183$ $r = 18.3\%$

b) $g(x) = 0.0123(1.0004)^x$

$a = 0.0123$ $b = 1.0004$ $r = 0.04\%$

c) $h(t) = 22(0.813)^t$

$a = 22$ $b = 0.813$ $r = -18.7\%$

d) $M(t) = 1145\left(\frac{3}{8}\right)^t$

$a = 1145$ $b = 3/8$ $r = -\frac{5}{8} = -62.5\%$

4. The value V of an investment in year t is given by $V = 2500(1.0325)^t$. Describe this investment to a person on the street in everyday English. (What was the initial investment, and what yearly interest rate is the investment earning?)

The investment was initially \$2500, then grew by 3.25% per year.

5. When you drink a cup of coffee, you ingest about 100mg of caffeine. Every hour, approximately 16% of the caffeine is removed from your body through metabolic processes. Write a formula for the amount of caffeine in your body t hours after you drink a cup. How much is left after 3 hours? About how long until there is only 10mg left in your body? (Use a calculator to help.) What is the one-day decay rate for caffeine in your body?

$r = -0.16$
$b = 1+r$
$= 0.84$

$A(t) = 100(0.84)^t$

$A(3) = 100(0.84)^3 = 59.27\ldots$

After 3 hours, you still have about 59 mg of caffeine.

$10 = 100(0.84)^t$

$\Downarrow$

$t \approx 13.2$ according to a graphical solving tool. It would take about 13 hours to get down to 10mg.

$A(24) = 100(0.84)^{24}$

$= 100(0.01523\ldots)$

$\Rightarrow b$ for a day is 0.01523

$\Rightarrow r$ for a day is -98.47%.