Solving Exponential Equations That Don't Require Logarithms

In these exercises, we will practice manipulating the symbols involved with exponential expressions, and solving certain equations where the variable is in the exponent. These will *not* require the use of logarithms, although for some we will "cheat" and use a graphing calculator.

1. Solve each of the following exponential equations. You may be able to solve some of these without a graphing calculator. For others, you will need to make a graph and use special solving tools on your calculator. Check all solutions by putting them back into the original equation.

(a) $4^x = 64$ $= 4^4$

$\Longrightarrow x = 4$

(b) $3 \cdot 5^x = 75$

$5^x = 25 = 5^2$

$\Longrightarrow x = 2$

(c) $2^{x^2} - 16 = 0$

$2^{x^2} = 16 = 2^4$

$\Longrightarrow x^2 = 4$

$\Longrightarrow x = 2$ or $x = -2$

(d) $2^{x^2-5x} = 2^{-6}$

$\Longrightarrow x^2 - 5x = -6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2$ or $x = 3$

(e) $3^x = 11$

Using calculator to solve graphically,

$x \approx 2.182\ldots$

(f) $4^{2x} + 2 \cdot 4^x = 3$

let $A = 4^x$

$4^{2x} + 2 \cdot 4^x - 3 = 0$

$(4^x)^2 + 2 \cdot 4^x - 3 = 0$

$A^2 + 2A - 3 = 0$

$(A+3)(A-1) = 0$

$\Longrightarrow A = -3$ or $A = 1$

$4^x = -3$ or $4^x = 1 \Longrightarrow x = 0$

(not possible)

(g) $5^{2x} - 1 = 0$

$5^{2x} = 1$

$\Longrightarrow 2x = 0$

$\Longrightarrow x = 0$

(h) $20^x - 2 \cdot 10^x + \cdot 5^x = 0$

$5^x(4^x - 2 \cdot 2^x + 1) = 0$

$\Longrightarrow 5^x = 0$ or $4^x - 2 \cdot 2^x + 1 = 0$

(not possible)

$(2^x)^2 - 2 \cdot 2^x + 1 = 0$

$\overset{A=2^x}{\Longleftrightarrow} A^2 - 2A + 1 = 0 \Longrightarrow (A-1)^2 = 0$

$\Downarrow$

$A = 1$

$\Downarrow$

$2^x = 1$

$\Downarrow$

$x = 0$

(i) $5^{\frac{2t+4}{2}} \cdot 2^{3t} = 40000$

$5^{t+2} \cdot (2^3)^t = 40000$

$5^t \cdot 5^2 \cdot 8^t = 40000$

$40^t = 1600 = 40^2$

$\Longrightarrow t = 2$

2. Use properties of exponents to write each of the following formulas in the form $f(t) = a\,b^t$. Leave your answers in exact form.

(a) $f(t) = 4^{t+1} \cdot 2^{3t}$

$= 4^t \cdot 4^1 (2^3)^t$

$= 4^t \cdot 4 \cdot 8^t$

$= 4 \cdot 32^t$

(b) $f(t) = 4^{\frac{t+1}{2}} \cdot 3^{2t}$.

$= (4^{1/2})^{t+1} (3^2)^t$

$= 2^{t+1} \cdot 9^t$

$= 2^t \cdot 2^1 \cdot 9^t$

$= 2 \cdot 18^t$

3. Myron and Winston have studied all of the exponential equations so far, and are trying to generalize their results. They begin with the equation

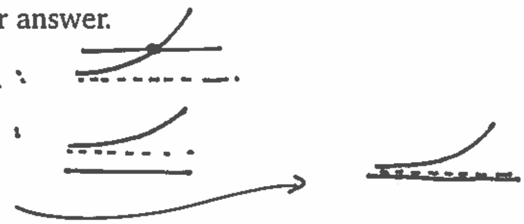
$$2^x = c$$

where c can be any real number. Help them decide if the following statements are true of false, if they are false, provide an example that supports your answer.

(a) If $c > 0$ then the equation has a solution. True :

(b) If $c < 0$ then the equation has a solution. False :

(c) If $c = 0$ then the equation has a solution. False



4. Imagine that you stand at your back door, facing into your back yard, holding a 1.5 ft-long magic telescope pointing straight up. For every foot that you step forward into your back yard, the telescope expands doubling its length.

(a) What is the length of the telescope when you are 1 ft from your back door? State your answers in ft.

1.5 ft, doubled, is 3 ft.

(b) What is the length of the telescope when you are 1 yd from your back door?

1 yd is 3 ft, so we have doubled 1.5 ft 3 times:

$$1.5 \times 2 \times 2 \times 2 = 1.5 \times 2^3 = 1.5 \cdot 8 = 12$$

It's 12 ft long.

(c) Given that the distance between the Earth and the Moon is approximately 1.329×10^9 ft, how far must you step for your telescope to be long enough to scrape the moon? (Set up an equation that represents this question, and then use a graphing calculator to help you solve it.)

number of ft you step forward

$$1.5 \cdot 2^n = 1.329 \times 10^9$$

Graphical solution Y1 = 1.5 * 2^X
Y2 = 1.329 * 10^9

Use Intersection Tool

$\Longrightarrow n = 29.7.$ If you walked 29.7 feet forward, your telescope could scrape the moon.