The Difference Quotient

In these exercises, we practice computing and simplifying an expression known as the *difference quotient*. At first, we will work with simpler building blocks to the difference quotient.

1. This exercise is just a warm-up. There is no difference quotient here. Let f be defined by $f(x) = 3x^2 + 2x - 4$.

a) Find $f(0)$.

$= 3(0)^2 + 2(0) - 4$

$= -4$

b) Find $f(1)$.

$= 3(1)^2 + 2(1) - 4$

$= 3 + 2 - 4$

$= 1$

c) Simplify $f(-x)$.

$= 3(-x)^2 + 2(-x) - 4$

$= 3x^2 - 2x - 4$

d) Simplify $-f(x)$.

$= -(3x^2 + 2x - 4)$

$= -3x^2 - 2x + 4$

e) Simplify $f(x+1)$.

$= 3(x+1)^2 + 2(x+1) - 4$

$= 3(x^2 + 2x + 1) + 2x + 2 - 4$

$= 3x^2 + 6x + 3 + 2x - 2$

$= 3x^2 + 8x + 1$

f) Simplify $f(2x)$.

$= 3(2x)^2 + 2(2x) - 4$

$= 3(4x^2) + 4x - 4$

$= 12x^2 + 4x - 4$

2. If $g(x) = x^2 + 3$, simplify $g(x+2) - g(x)$.

$= (x+2)^2 + 3 - (x^2 + 3)$

$= x^2 + 4x + 4 + 3 - x^2 - 3$

$= 4x + 4$

3. If $k(x) = \frac{3}{x}$, simplify $k(x+2) - k(x)$.

$= \frac{3}{x+2} - \frac{3}{x}$

$= \frac{3x}{x(x+2)} - \frac{3(x+2)}{x(x+2)}$

$= \frac{3x}{x(x+2)} - \frac{3x+6}{x(x+2)}$

$= \frac{-6}{x(x+2)}$

4. For each of these functions, find and simplify the *difference quotient*. For a function f, the difference quotient of f is $\frac{f(x+h)-f(x)}{h}$.

a) $f(x)=4x+3$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{4(x+h)+3-(4x+3)}{h}$$
$$=\frac{4x+4h+3-4x-3}{h}$$
$$=\frac{4h}{h}$$
$$=4, \quad h\neq 0$$

b) $g(x)=x^2-4$

$$\frac{g(x+h)-g(x)}{h}=\frac{(x+h)^2-4-(x^2-4)}{h}$$
$$=\frac{x^2+2xh+h^2-4-x^2-4}{h}$$
$$=\frac{2xh+h^2}{h}$$
$$=\frac{h(2x+h)}{h}$$
$$=2x+h, \quad h\neq 0$$

c) $p(x)=2x^2-3x+1$

$$\frac{p(x+h)-p(x)}{h}$$
$$=\frac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h}$$
$$=\frac{2(x^2+2xh+h^2)-3x-3h+1-2x^2+3x-1}{h}$$
$$=\frac{2x^2+4xh+2h^2-3h-2x^2}{h}$$
$$=\frac{4xh+2h^2-3h}{h}$$
$$=\frac{h(4x+2h-3)}{h}$$
$$=4x+2h-3, \quad h\neq 0$$

d) $k(x)=\frac{1}{x+3}$

$$\frac{k(x+h)-k(x)}{h}=\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}$$
$$=\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}\cdot\frac{x+h+3}{x+h+3}$$
$$=\frac{1-\frac{x+h+3}{x+3}}{h(x+h+3)}\cdot\frac{x+3}{x+3}$$
$$=\frac{x+3-(x+h+3)}{h(x+h+3)(x+3)}$$
$$=\frac{x+3-x-h-3}{h(x+h+3)(x+3)}$$
$$=\frac{-h}{h(x+h+3)(x+3)}$$
$$=\frac{-1}{(x+h+3)(x+3)}, \quad h\neq 0$$