

Concavity

In these exercises, we will examine the concavity of various functions presented in different ways.

1. Does the function provided by each of these tables appear to be concave up, concave down, or neither? Include a brief explanation with your answer.

a)

	x	$f(x)$	
+2	4	12	-1
	6	11	
+2	8	9	-2
	10	6	

decreasing at
an increasing rate,
so concave down

b)

	x	$g(x)$	
+3	14	7	+4
	17	11	
+3	20	15	+4
	23	19	

increasing
at a constant
rate (slope $\frac{4}{3}$)
so no concavity

c)

	x	$h(x)$	
+1	3	1.2	+0.02
	4	1.22	
+1	5	1.26	+0.04
	6	1.34	

increasing at
an increasing rate,
so concave up

2. Sketch the graph of a function f that is concave up but always decreasing.



3. When money is deposited in the bank, the amount of money increases slowly at first. As the size of the account increases, the amount of money increases more rapidly, since the account is earning interest on the new interest as well as on the original deposit.

- a) Is the function being described here increasing, decreasing, both, or neither? b) Is it concave up or concave down, at times both, or neither?

increasing

concave up

4. When a communicable disease breaks out in an isolated village, at first only a few people have it, and it spreads slowly. As more people become sick, the disease can spread faster, and the number of sick people grows faster. Later, when a majority of people are already sick, the number of sick people stops growing so fast and slows down, since there aren't that many uninfected people left.



- a) Is the function being described here increasing, decreasing, both, or neither? b) Is it concave up or concave down, at times both, or neither?

increasing

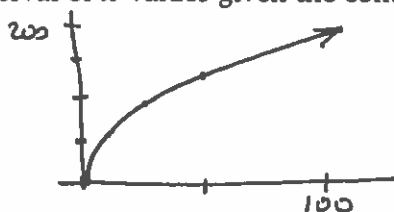
concave up at first,
then concave down

5. The swimming speed of a salmon U (in $\frac{\text{cm}}{\text{s}}$) is related to the length of the salmon l (in cm), according to

$$U = f(l)$$

$$f(l) = 19.5\sqrt{l}$$

- (a) Use your calculator to graph this function. Choose a viewing window with a reasonable interval of x -values given the context of the problem.



- (b) Is the function increasing or decreasing? Concave up or concave down?

increasing

concave down

- (c) Imagine four salmon. Two of them have the lengths: A is 50 cm and B is 55 cm. B, being larger, swims faster than A. The other two salmon are larger in length: C is 80 cm and D is 85 cm. Is the amount by which B swims faster than A *more* or *less* than the amount by which D swims faster than C? Don't calculate anything—just use your understanding of concavity to answer.

and increasing

Since f is concave down, it is increasing at an increasing decreasing rate. So the difference $f(85) - f(80)$ is smaller than the difference $f(55) - f(50)$. So the amount by which B swims faster than A is more than the amount by which D swims faster than C.