

Composition of Functions

In these exercises, we will practice composing two functions together, and also see how this is useful in some contexts.

1. Let $f(x) = 5x^2$ and $g(x) = x - 3$.

a) Find and simplify $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-3) \\ &= 5(\quad)^2 \\ &= 5(x-3)^2 \\ &= 5(x^2 - 6x + 9) \\ &= 5x^2 - 30x + 45 \end{aligned}$$

c) Find and simplify $(f \circ g)(x)$.

it's the same as part a),
just using different
notation

e) Find and simplify $(f \circ f)(1)$.

$$\begin{aligned} (f \circ f)(1) &= f(f(1)) \\ &= f(5) \quad (\text{since } f(1) = 5(1)^2 = 5) \\ &= 5(5)^2 \\ &= 125 \end{aligned}$$

b) Find and simplify $g(f(2))$.

$$\begin{aligned} g(f(2)) &= g(20) \quad (\text{since } f(2) = 5(2)^2 = 20) \\ &= 20 - 3 \\ &= 17 \end{aligned}$$

d) Find and simplify $g(g(x))$.

$$\begin{aligned} g(g(x)) &= g(x-3) \\ &= (\quad) - 3 \\ &= (x-3) - 3 \\ &= x-6 \end{aligned}$$

Alternatively, g "subtracts 3".
So $g \circ g$ subtracts 3
then subtracts 3
again. So it
subtracts 6,
so $g(g(x)) = x-6$

f) Find and simplify $f(g(7))$.

$$\begin{aligned} f(g(7)) &= f(4) \\ &= 5(4)^2 \\ &= 80 \end{aligned}$$

since $g(7) = 4$

2. Suppose that $\ell = g(t)$, where ℓ is the length (in centimeters) of a plant leaf at time t (in days) after the leaf sprouted. Also, let $s = f(\ell)$, where s is the amount of sugar per day (in grams/day) that a leaf produces when it is ℓ cm long.

(a) What is the meaning of $f(g(t))$? That is, give an English sentence that provides context for $f(g(t))$; something like " $f(g(t))$ is the _____ after _____."

$f(g(t))$ is the amount of sugar per day (in grams/day) that a leaf produces after t days.

(b) What units does $f(g(t))$ have?

grams/day (because that's what f outputs)