

Adding, Subtracting, Multiplying, and Dividing Functions, and the Resulting Domain

In these exercises, we combine two functions using addition, subtraction, multiplication, and division to obtain a new function. This may feel deceptively simple; it is actually a pretty complicated idea. Then the more challenging part will be writing down the domain of the new function.

1. In these problems you are given formulas for two functions f and g . (a) Find and simplify a formula for the indicated combined function. (b) Find the domain for the indicated combined function. It is not the correct approach to do this by looking at the formula for the indicated combined function. Instead you should think about what f 's domain is and what g 's domain is, and combine them appropriately.

a) $f(x) = 3x + 4$ $g(x) = 2x - 3$

Investigate $f + g$.

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= 3x + 4 + 2x - 3 \\ &= 5x + 1\end{aligned}$$

f 's domain is $(-\infty, \infty) \Rightarrow (f+g)$'s domain is $(-\infty, \infty)$
 g 's domain is $(-\infty, \infty)$

c) $f(x) = \frac{1}{x}$ $g(x) = \frac{1}{x(x+1)}$

Investigate $f - g$.

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= \frac{1}{x} - \frac{1}{x(x+1)} \\ &= \frac{x+1}{x(x+1)} - \frac{1}{x(x+1)} \\ &= \frac{x}{x(x+1)} = \frac{1}{x+1}\end{aligned}$$

f 's domain is $(-\infty, 0) \cup (0, \infty)$
 g 's domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$
 so $(f-g)$'s domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

2. Here are tables for two functions of f and g .

$U(-1, 0)$
 $U(0, \infty)$

x	$f(x)$	x	$g(x)$
2	1	1	2
3	0	2	3
5	3	3	2
7	4	5	0
8	5	8	-1
11	4	11	2
12	3	12	6

b) $f(x) = \sqrt{x+1}$ $g(x) = \sqrt{x-1}$

Investigate $f \cdot g$.

$$\begin{aligned}(f \cdot g)(x) &= \sqrt{x+1} \cdot \sqrt{x-1} \\ &= \sqrt{(x+1)(x-1)} \\ &= \sqrt{x^2-1}\end{aligned}$$

f 's domain is $[-1, \infty)$ $\Rightarrow (f \cdot g)$'s domain is $[1, \infty)$
 g 's domain is $[1, \infty)$

d) $f(x) = \frac{x^2-4}{x-3}$ $g(x) = \frac{x-2}{x+3}$

Investigate f/g .

$$\begin{aligned}(f/g)(x) &= \frac{f(x)}{g(x)} = \frac{\frac{x^2-4}{x-3}}{\frac{x-2}{x+3}} = \frac{x^2-4}{x-3} \cdot \frac{x+3}{x-2} \\ &= \frac{(x+2)(x-2)}{x-3} \cdot \frac{x+3}{x-2} = \frac{(x+2)(x+3)}{x-3}\end{aligned}$$

f 's domain is $(-\infty, 3) \cup (3, \infty)$.
 g 's domain is $(-\infty, -3) \cup (-3, \infty)$.
 So f/g 's domain is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.
 $g(x) = 0$ when $x = 2$.

- (a) Find $(f-g)(3) = f(3) - g(3) = 0 - 2 = -2$
 (b) Find $(f \cdot g)(11) = f(11) \cdot g(11) = 4 \cdot 2 = 8$
 (c) Find $(f-g)(7)$. is undefined since 7 is not in g 's domain
 (d) Find $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{1}{3}$
 (e) Find $\left(\frac{f}{g}\right)(5)$. is undefined since $g(5) = 0$

3. Combining functions without having their formulas:

- a) Suppose that $I(x)$ represents the income of an individual in year x before taxes, and $T(x)$ represents the individual's tax bill in year x . Determine the name of a function that represents the person's *net* income (income after taxes) in year x .

$$I - T$$

- b) Suppose that $P(x)$ represents the *fraction* of an individual's income spent on health care in year x , and $I(x)$ represents income in year x . Determine the name of a function that represents the total health care expenditures for that individual in year x .

$$I \cdot P$$

4. Suppose that we put a population of 100 female insects in a box with temperature $x^\circ\text{F}$, and that as a result after one week, there are $F(x)$ living females still in the box. Some females will survive in temperatures ranging from 40°F to 110°F , but outside of that range all of the female insects will die. As an example, if $F(82) = 91$, we are saying that when you put 100 female insects in a box for one week at a temperature of 82°F , that 91 will survive.

We also do the same thing with 100 males in a different box. Let $M(x)$ be the number of living males after one week. Some males survive in temperatures ranging from 50°F to 115°F . After one week, we combine the boxes and let $R(x)$ be the *ratio* of living females to living males.

- a) What is the domain of F ?

$$(-\infty, \infty)$$

Actually, whatever the absolute zero temp is in $^\circ\text{F}$: $[-459.67, \infty)$

- c) What is the domain of M ?

$$(-\infty, \infty)$$

or

$$[-459.67, \infty)$$

- e) What is the domain of R ?

$$R(x) = \frac{F(x)}{M(x)} \quad \text{so } R \text{'s}$$

$$\text{domain is } \underbrace{(-\infty, \infty)}_{F \text{'s domain}} \cap \underbrace{(-\infty, \infty)}_{M \text{'s domain}} \cap [50, 115] \text{ where } M(x) \neq 0$$

$$\text{domain is } [50, 115]$$

- b) For what x values does $F(x) = 0$?

$$F(x) = 0 \text{ when } x \text{ is in } (-\infty, 40) \cup (110, \infty)$$

$$[-459.67, 40) \cup (110, \infty)$$

- d) For what x values does $M(x) = 0$?

$$M(x) = 0 \text{ when } x \text{ is in } (-\infty, 50) \cup (115, \infty)$$

$$\text{or } [-459.67, 50) \cup (115, \infty)$$