

Inverses of Functions

In these exercises, we will take a given (invertible) function and represent its inverse function with a formula, with a table, verbally, and graphically.

- Find formulas for the inverse functions of these functions. As a reminder, formulas for functions can use anything for the dummy variable. For instance, the formulas $q(x) = x^2$, $q(y) = y^2$, and $q(A) = A^2$ all say the same thing: q is a function that squares its input. This might clear up some confusion about how to write formulas for the inverse functions below.

a) $y = f(x)$, where $f(x) = 2x - 9$

b) $P = g(t)$, where $g(t) = \sqrt[3]{30t} + 1$

c) $C = h(w)$, where $h(w) = \frac{w}{3 + 2w}$

- Use the table to make a table of values for f^{-1} .

x	2	4	6	8	10
$f(x)$	8	2	4	1	10

- The perimeter of a square of side s (measured in cm) is given by $P = f(s)$, where $f(s) = 4s$, and P is measured in cm. Find the following, and then interpret what you have found (using everyday-language):

a) $f(3)$

b) $f^{-1}(20)$

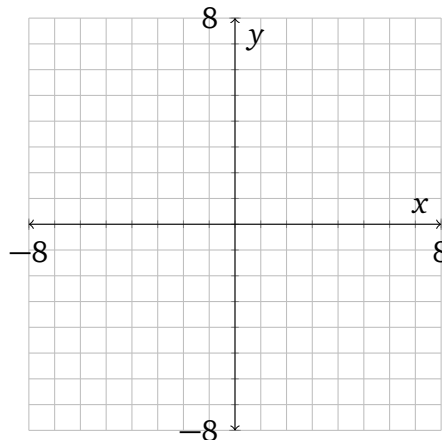
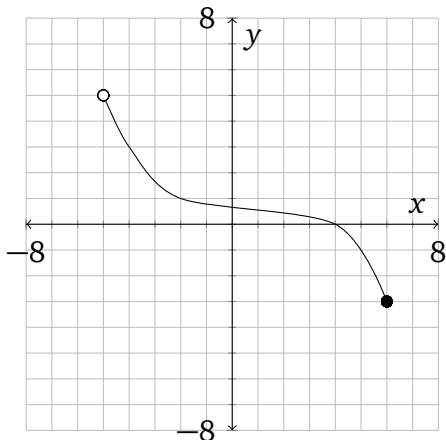
c) $f^{-1}(P)$

4. The gross domestic product (GDP) of the US is given by $G(t)$, where t is the number of years since 1990 and the units of G are billions of dollars.

a) What does $G(11)$ represent?

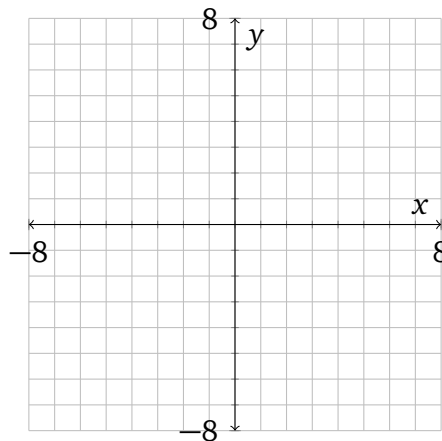
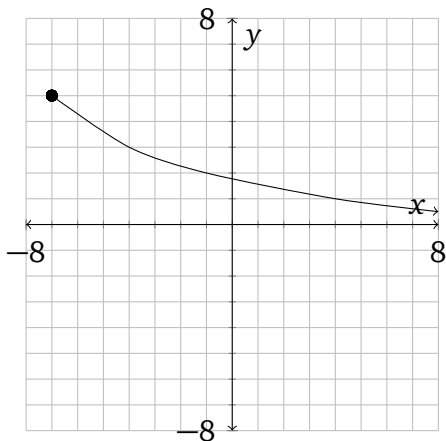
b) What does $G^{-1}(9873)$ represent?

5. Let $y = f(x)$, where f is defined by the graph below.



Graph the curve $y = f^{-1}(x)$. Use the specific input-output pairs that you found in part (a), but plot them appropriately for a graph of f^{-1} .

6. Plot a graph of g^{-1} if g is defined by the graph below.



7. (Challenging) Use your calculator to graph both

$$f(x) = \frac{1}{2}x^3 + \frac{3}{2^{1/3}}x \text{ and } g(x) = \left(\sqrt{x^2 + 4} + x\right)^{1/3} - \left(\sqrt{x^2 + 4} - x\right)^{1/3}.$$

Then show that f and g are inverse functions of each other. To do this, you will need a lot of space on separate paper to correctly simplify $f(g(x))$ and $g(f(x))$. This exercise is a challenge to you that you can succeed with if you carefully apply basic algebra. It is too challenging for an exam though.