**SUMMARY OF FUNCTION TRANSFORMATIONS**

The graph of \( y = Af(B(x + h)) + k \) is a transformation of the graph of \( y = f(x) \). The transformations can be done in the following order:

- **A**: The function stretches or compresses vertically by a factor of \(|A|\). If \( A \) is negative, the function also reflects across the \( x \)-axis.
- **B**: The function stretches or compresses horizontally by a factor of \(|\frac{1}{B}|\). If \( B \) is negative, the function also reflects across the \( y \)-axis.
- **h**: The function shifts horizontally by \( h \) units. If \( h > 0 \), the function shifts left. If \( h < 0 \), the function shifts right.
- **k**: The function shifts vertically by \( k \) units. If \( k > 0 \), the function shifts up. If \( k < 0 \), the function shifts down.

**Other orderings are possible; this ordering will work in all cases.**

Graph the transformations below by doing the following on graphing paper:

- Graph the basic function used in this transformation. (Example: \( f(x) = x^2 \)). Use your Library of Functions Handout if necessary.
- State the series of transformations and the order in which they occur.
- Graph the transformation.
- Check your graph using the interactive GeoGebra applet located on the course webpage. You will also want to check your graph using a few of the input/output pairs.
- Hand-sketched examples are located under “Contents.”
- Full solutions to the exercises below are given in the next few pages.

**Section I: Horizontal and Vertical Shifts**

- (a) \( g_1(x) = (x - 5)^2 + 1 \)
- (b) \( g_2(x) = \sqrt{x+4} + 2 \)
- (c) \( g_3(x) = (x+1)^3 - 2 \)
- (d) \( g_4(x) = \frac{1}{x-2} + 2 \)
- (e) \( g_5(x) = |x-8| + 6 \)

**Section II: Horizontal and Vertical Stretches and Reflections**

- (a) \( g_1(x) = -\sqrt{x} \)
- (b) \( g_2(x) = \sqrt{-x} \)
- (c) \( g_3(x) = 4x^3 \)
- (d) \( g_4(x) = -2x^2 \)
- (e) \( g_5(x) = |5x| \)
- (f) \( g_6(x) = -2\sqrt{-x} \)

**Section III: General Function Transformations**

- (a) \( g_1(x) = 3(x-2)^2 + 5 \)
- (b) \( g_2(x) = -(x+1)^3 - 3 \)
- (c) \( g_3(x) = |2x| - 3 \)
- (d) \( g_4(x) = \sqrt{-x} + 4 \)
- (e) \( g_5(x) = \frac{2}{x} + 5 \)
- (f) \( g_6(x) = 4\sqrt{-2(x+1)} + 3 \)
1. **Horizontal and Vertical Shifts**

1. Transforming \( f(x) = x^2 \) into \( g_1(x) = (x - 5)^2 + 1 \):

The graph of \( y = g_1(x) \) is in Figure 1. It is obtained by the following transformations:

   (a) \( h = -5 \): Shift 5 units right
   (b) \( k = 1 \): Shift 1 unit up

\[ \text{Figure 1} \]

2. Transforming \( f(x) = \sqrt{x} \) into \( g_2(x) = \sqrt{x + 4} + 2 \):

The graph of \( y = g_2(x) \) is in Figure 2. It is obtained by the following transformations:

   (a) \( h = 4 \): Shift 4 units left
   (b) \( k = 2 \): Shift 2 units up

\[ \text{Figure 2} \]
3. Transforming \( f(x) = x^3 \) into \( g_3(x) = (x + 1)^3 - 2 \):

The graph of \( y = g_3(x) \) is in Figure 3. It is obtained by the following transformations:

(a) \( h = 1 \): Shift 1 units left

(b) \( k = 2 \): Shift 2 units down

4. Transforming \( f(x) = \frac{1}{x} \) into \( g_4(x) = \frac{1}{x - 2} + 2 \):

The graph of \( y = g_4(x) \) is in Figure 4. It is obtained by the following transformations:

(a) \( h = -2 \): Shift 2 units right

(b) \( k = 2 \): Shift 2 units up
5. Transforming $f(x) = |x|$ into $g_5(x) = |x - 8| + 6$:

The graph of $y = g_5(x)$ is in Figure 5. It is obtained by the following transformations:

(a) $h = -8$: Shift 8 units right
(b) $k = 6$: Shift 6 units up

![Figure 5]
6. Transforming \( f(x) = \sqrt{x} \) into \( g_1(x) = -\sqrt{x} \):

The graph of \( y = g_1(x) \) is in Figure 6. It is obtained by the following transformations:

(a) \( A = -1 \): Reflect across the \( x \)-axis

7. Transforming \( f(x) = \sqrt{x} \) into \( g_2(x) = \sqrt{-x} \):

The graph of \( y = g_2(x) \) is in Figure 7. It is obtained by the following transformations:

(a) \( B = -1 \): Reflect across the \( y \)-axis
8. Transforming $f(x) = x^3$ into $g_3(x) = 4x^3$:

The graph of $y = g_3(x)$ is in Figure 8. It is obtained by the following transformations:

(a) $A = 4$ Stretch vertically by a factor of 4

![Figure 8](image)

9. Transforming $f(x) = x^2$ into $g_4(x) = -2x^2$:

The graph of $y = g_4(x)$ is in Figure 9. It is obtained by the following transformations:

(a) $A = -2$: Reflect across the horizontal axis AND stretch vertically by a factor of 2

![Figure 9](image)
10. Transforming $f(x) = |x|$ into $g_5(x) = 5|x|$: 

The graph of $y = g_5(x)$ is in Figure 10. It is obtained by the following transformations:

(a) $B = 5$: Compress horizontally by a factor of $\frac{1}{5}$.

![Figure 10](image)

11. Transforming $f(x) = \sqrt{x}$ into $g_6(x) = -2\sqrt{-x}$: 

The graph of $y = g_6(x)$ is in Figure 11. It is obtained by the following transformations:

(a) $A = -2$ Reflect across the x-axis AND stretch vertically by a factor of 2
(b) $B = -1$ Reflect across the y-axis

![Figure 11](image)
3. General Function Transformations

12. Transforming \( f(x) = x^2 \) into \( g_1(x) = 3(x - 2)^2 + 5 \):

The graph of \( y = g_1(x) \) is in Figure 12. It is obtained by the following transformations:

(a) \( A = 3 \): Stretch vertically by a factor of 3
(b) \( h = -2 \): Shift 2 units right
(c) \( k = 5 \): Shift 5 units up

![Figure 12](image)

13. Transforming \( f(x) = x^3 \) into \( g_2(x) = -(x + 1)^3 - 3 \):

The graph of \( y = g_2(x) \) is in Figure 13. It is obtained by the following transformations:

(a) \( A = -1 \): Reflect across the \( x \)-axis
(b) \( h = -1 \): Shift 1 unit left
(c) \( k = -3 \): Shift 3 units down

![Figure 13](image)
14. Transforming \( f(x) = |x| \) into \( g_3(x) = |2x| - 3 \):

The graph of \( y = g_3(x) \) is in Figure 14. It is obtained by the following transformations:

(a) \( B = 2 \): Compress horizontally by a factor of \( \frac{1}{2} \)
(b) \( k = -3 \): Shift 3 units down

![Figure 14](chart.png)

15. Transforming \( f(x) = \sqrt{x} \) into \( g_4(x) = \sqrt{-x} + 4 \):

The graph of \( y = g_4(x) \) is in Figure 15. It is obtained by the following transformations:

(a) \( B = -1 \): Reflect across the \( y \)-axis
(b) \( k = 4 \): Shift 4 units up

![Figure 15](chart.png)
16. Transforming $f(x) = \frac{1}{x}$ into $g_5(x) = \frac{2}{x} + 5$:

The graph of $y = g_5(x)$ is in Figure 16. It is obtained by the following transformations:

(a) $A = 2$: Stretch vertically by a factor of 2
(b) $k = 5$: Shift 5 units up

![Figure 16](image)

17. Transforming $f(x) = \sqrt{x}$ into $g_6(x) = 4\sqrt{-2(x + 1) + 3}$:

The graph of $y = g_6(x)$ is in Figure 17. It is obtained by the following transformations:

(a) $A = 4$: Stretch vertically by a factor of 4
(b) $B = -2$: Compress horizontally by a factor of $\frac{1}{2}$ AND reflect across the $y$-axis
(c) $h = 1$: Shift 1 unit left
(d) $k = 3$: Shift 3 units up

![Figure 17](image)