Supplemental Solutions for the Rates of Change Lab

Exercise 1.1

E1.1.1 The difference quotient for z is:

$$\begin{aligned}\frac{z(x+h)-z(x)}{h} &= \frac{\left[2+4(x+h)-(x+h)^2\right]-\left[2+4x-x^2\right]}{h}\\ &= \frac{2+4x+4h-x^2-2xh-h^2-2-4x+x^2}{h}\\ &= \frac{4h-2xh-h^2}{h}\\ &= \frac{h(4-2x-h)}{h}\\ &= 4-2x-h;\ \text{for } h\neq 0\end{aligned}$$

E1.1.2 The rise between the two points is 9 and the run is 3, so the slope between the two points is given by $\frac{9}{3}=3$.

Using the difference quotient, if we let $x=-1$ and $h=3$ we get:

$$\begin{aligned}4-2x-h &= 4-2(-1)-3\\ &= 3 \ \checkmark\end{aligned}$$

E1.1.3 $$\begin{aligned}\frac{z(4+h)-z(4)}{h} &= 4-2(4)-h\\ &= -4-h\end{aligned}$$

Table E1.1K: $y=\frac{z(4+h)-z(4)}{h}$

h	y
–0.1	–3.9
–0.01	–3.99
–0.001	–3.999
0.001	–4.001
0.01	–4.01
0.1	–4.01

E1.1.4 The values are converging on -4.

E1.1.5

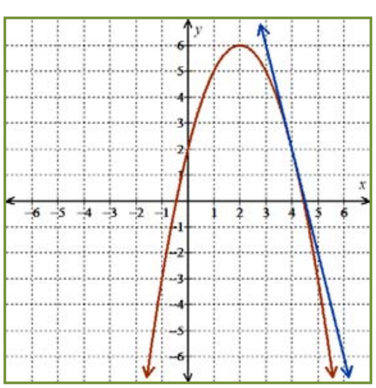

Figure E1.1K: $y=2+4x-x^2$

Exercise 1.2

E1.2.1

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\left[3-7(x+h)\right]-\left[3-7x\right]}{h}\\ &= \frac{3-7x-7h-3+7x}{h}\\ &= \frac{-7h}{h}\\ &= -7;\ \text{for}\ h\neq 0\end{aligned}$$

E1.2.2

$$\begin{aligned}\frac{g(x+h)-g(x)}{h} &= \frac{\dfrac{7}{x+h+4}-\dfrac{7}{x+4}}{h}\\ &= \frac{\dfrac{7}{x+h+4}\cdot\dfrac{x+4}{x+4}-\dfrac{7}{x+4}\cdot\dfrac{x+h+4}{x+h+4}}{\dfrac{h}{1}}\\ &= \frac{7x+28-7x-7h-28}{(x+h+4)(x+4)}\cdot\frac{1}{h}\\ &= \frac{-7h}{(x+h+4)(x+4)h}\\ &= \frac{-7}{(x+4)(x+h+4)};\ \text{for}\ h\neq 0\end{aligned}$$

E1.2.3

$$\begin{aligned}\frac{z(x+h)-z(x)}{h} &= \frac{\pi-\pi}{h}\\ &= \frac{0}{h}\\ &= 0;\ \text{for}\ h\neq 0\end{aligned}$$

E1.2.4

$$\begin{aligned}\frac{s(t+h)-s(t)}{h} &= \frac{\left[(t+h)^3-(t+h)-9\right]-\left[t^3-t-9\right]}{h}\\ &= \frac{t^3+3t^2h+3th^2+h^3-t-h-9-t^3+t+9}{h}\\ &= \frac{3t^2h+3th^2+h^3-h}{h}\\ &= \frac{\left(3t^2+3th+h^2-1\right)h}{h}\\ &= 3t^2+3th+h^2-1;\ \text{for}\ h\neq 0\end{aligned}$$

E1.2.5

$$\frac{k(t+h)-k(t)}{h}=\frac{\frac{(t+h-8)^2}{t+h}-\frac{(t-8)^2}{t}}{h}$$

$$=\frac{\frac{t^2+th-8t+th+h^2-8h-8t-8h+64}{t+h}-\frac{t^2-8t-8t+64}{t}}{h}$$

$$=\frac{\frac{t^2+2th-16t+h^2-16h+64}{t+h}\cdot\frac{t}{t}-\frac{t^2-16t+64}{t}\cdot\frac{t+h}{t+h}}{\frac{h}{1}}$$

$$=\frac{t^3+2t^2h-16t^2+th^2-16th+64t-\left[t^3+t^2h-16t^2-16th+64t+64h\right]}{t(t+h)}\cdot\frac{1}{h}$$

$$=\frac{t^3+2t^2h-16t^2+th^2-16th+64t-t^3-t^2h+16t^2+16th-64t-64h}{t(t+h)}\cdot\frac{1}{h}$$

$$=\frac{t^2h+th^2-64h}{t(t+h)h}$$

$$=\frac{(t^2+th-64)h}{t(t+h)h}$$

$$=\frac{t^2+th-64}{t(t+h)};\ \text{for }\ h\neq 0$$

Exercise 1.3

E1.3.1

$$\frac{s(t+h)-s(t)}{h}=\frac{\left[150+60(t+h)-16(t+h)^2\right]-\left[150+60t-16t^2\right]}{h}$$

$$=\frac{150+60t+60h-16t^2-32th-16h^2-150-60t+16t^2}{h}$$

$$=\frac{60h-32th-16h^2}{h}$$

$$=\frac{(60-32t-16h)h}{h}$$

$$=60-32t-16h\ \ \text{for } h\neq 0$$

E1.3.2 Letting $t=4$ and $h=0.2$ we have (from the difference quotient):

$$60-32t-16h=60-32(4)-16(0.2)$$
$$=-71.2$$

Checking … $$\frac{s(4.2)-s(4)}{4.2-4}=\frac{119.76-134}{.2}$$ (Phew!)

$$=-71.2$$

E1.4.1 $\dfrac{v(7.5)-v(0)}{7.5\,\text{s}-0\,\text{s}}=-13\tfrac{1}{3}\ \dfrac{\text{ft/s}}{\text{s}}$

This value tells us that during the first 7.5 seconds of descent, the average rate of change in the coaster's velocity is $-13\tfrac{1}{3}\ \dfrac{\text{ft/s}}{\text{s}}$. In other words, ***on average***, with each passing second the velocity is $13\tfrac{1}{3}$ ft/s less than it was the preceding second. We could also say that the average acceleration experienced by the coaster over the first 7.5 seconds of descent is $-13\tfrac{1}{3}\ \dfrac{\text{ft/s}}{\text{s}}$.

E1.4.2 $\dfrac{h(3)-h(1.5)}{3\,\text{s}-1.5\,\text{s}}=0\ \text{m/s}$

This value tells us that the average velocity experienced by the ball between the 1.5 second of play and the third second of play is $0\ \text{m/s}$. Please note that this does not imply that the ball doesn't move; it simply means that the ball is at the same elevation at the two times.

E1.4.3 $\dfrac{P(29)-P(1)}{29\,\text{swing}-1\,\text{swing}}=-0.1\ \text{s/swing}$

This value tells us that between the first swing and the 29th swing the average rate of change in the pendulum's period is $-0.1\ \text{s/swing}$. In other words, ***on average***, with each passing swing the period decreases by a tenth of a second.

E1.4.4 $\dfrac{h(120)-h(60)}{120\,\text{s}-60\,\text{s}}=.13\,\dfrac{\text{mph/s}}{\text{s}}$

This value tells us that during the second minute of flight, the average rate of change in the rocket's acceleration is $.13\,\dfrac{\text{mph/s}}{\text{s}}$. In other words, ***on average***, with each passing second the acceleration is $.13$ mph/s more than it was the preceding second. You might have noticed that since the acceleration function is linear, the rate of change in the acceleration is constant. That is, with each passing second the acceleration ***actually is*** $.13$ mph/s more than it was the preceding second.