Solutions to the Supplemental Exercises for the Limits and Continuity Lab

Exercise 2.1

Table	Limit	Existence?	Comments
E2.2	$\lim_{x \to \infty} f(x) = 0$	Yes	The relevant pattern in the y column is the powers of 10. The first nonzero digit is moving farther and farther to the right of the decimal point.
E2.3	$\lim_{t \to \frac{1}{3}^-} z(t) = \frac{2}{3}$	Yes	You should definitely recognize decimals approaching common fractions. Good catch if you noted the t was approaching $\frac{1}{3}$ only from the left.
E2.4	$\lim_{\theta \to -1^+} g(\theta) = 3,000,000$	Yes	The last entry in the output column is the same in these two tables. Hopefully you recognized that you need to look at the pattern in the output, not just the last entry.
E2.5	$\lim_{t \to \frac{7}{9}^+} T(t) = \infty$	No	Recognizing that t approaches $\frac{7}{9}$ probably requires some guessing and checking. Stay focused when determining "from the left" or "from the right;" that can get tricky … especially when the numbers are negative.

Exercise 2.2

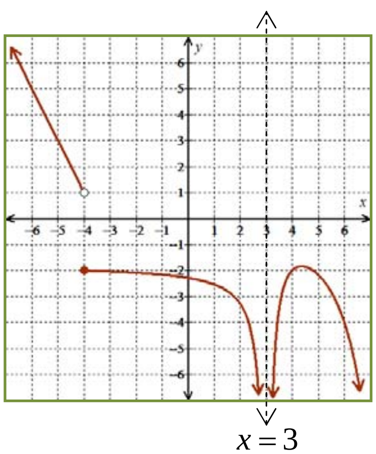

Figure E2.1K: f

Exercise 2.3

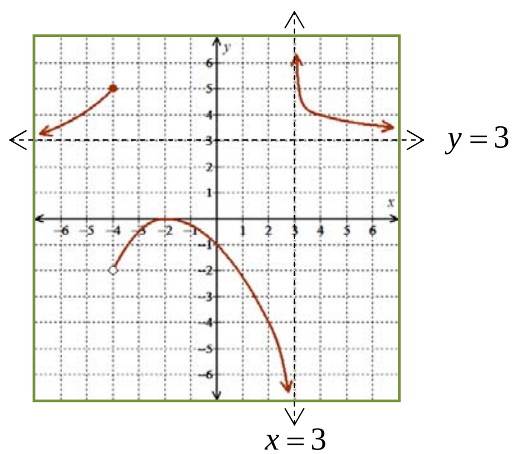

Figure E2.2K: f

Exercise 2.4

f is discontinuous from both directions at 0 and the discontinuity is not removable.

f is discontinuous at 3 although it is continuous from the left at 3. The discontinuity is not removable. (The problem is that the limits are different from the left and right of 3.)

f is discontinuous from both directions at 4 but the discontinuity is removable ($\lim\limits_{x\to 4} f(x)=1$).

f is discontinuous from both directions at 2π but the discontinuity is removable ($\lim\limits_{x\to 2\pi} f(x)=1$).

Exercise 2.5

Because the two formulas are polynomials, the only number where continuity is at issue is 3. The top formula is used at 3, so $g(3)$ is defined regardless of the value of k. Basically all we need to do is ensure that the limits from the left and the right of 3 are both equal to $g(3)$. We have:

$$g(3)=9+2k\,,\ \lim_{t\to 3^+} g(t)=9+2k\text{ , and }\lim_{t\to 3^-} g(t)=9-4k$$

So g will be continuous at 3 (and, consequently, over $(-\infty,\infty)$) if and only if $9+2k=9-4k$. This gives us $k=0$. Doh! Turns out the g isn't piece-wise at all, it's simply the parabolic function $g(t)=t^2$.

Exercise 2.6

E2.6.1 $\lim\limits_{x\to 4^-}\left(5-\dfrac{1}{x-4}\right)=\infty$ This limit does **not** exit.

E2.6.2

$$\begin{aligned}\lim_{x\to\infty}\frac{e^{2/x}}{e^{1/x}} &= \lim_{x\to\infty} e^{1/x} \\ &= e^{\lim\limits_{x\to\infty}\frac{1}{x}} && \text{LL A6} \\ &= e^0 && \text{LL R3} \\ &= 1\end{aligned}$$

This limit **does** exist.

E2.6.3

$$\begin{aligned}\lim_{x\to 2^+}\frac{x^2-4}{x^2+4} &= \frac{\lim\limits_{x\to 2^+}(x^2-4)}{\lim\limits_{x\to 2^+}(x^2+4)} && \text{LL A5} \\ &= \frac{\lim\limits_{x\to 2^+}x^2-\lim\limits_{x\to 2^+}4}{\lim\limits_{x\to 2^+}x^2+\lim\limits_{x\to 2^+}4} && \text{LL A1 and A2} \\ &= \frac{\left(\lim\limits_{x\to 2^+}x\right)^2-\lim\limits_{x\to 2^+}4}{\left(\lim\limits_{x\to 2^+}x\right)^2+\lim\limits_{x\to 2^+}4} && \text{LL A6} \\ &= \frac{2^2-4}{2^2+4} && \text{LL R1 and R2} \\ &= 0\end{aligned}$$

This limit **does** exist.

E2.6.4

$$\lim_{x\to 2^+}\frac{x^2-4}{x^2-4x+4}=\lim_{x\to 2^+}\frac{(x+2)(x-2)}{(x-2)(x-2)}$$

$$=\lim_{x\to 2^+}\frac{(x+2)}{(x-2)} \qquad \text{LL A7}$$

$$=\infty$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{0}{0}$.

This limit does **not** exist.

E2.6.5

$$\lim_{x\to\infty}\frac{\ln(x)+\ln(x^6)}{7\ln(x^2)}=\lim_{x\to\infty}\frac{\ln(x)+6\ln(x)}{7\cdot 2\ln(x)}$$

$$=\lim_{x\to\infty}\frac{7\ln(x)}{14\ln(x)}$$

$$=\lim_{x\to\infty}\frac{1}{2} \qquad \text{LL A7}$$

$$=\frac{1}{2} \qquad \text{LL R2}$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{\infty}{\infty}$.

This limit **does** exist.

E2.6.6

$$\lim_{x\to-\infty}\frac{3x^3+2x}{3x-2x^3}=\lim_{x\to-\infty}\left(\frac{3x^3+2x}{3x-2x^3}\cdot\frac{1/x^3}{1/x^3}\right)$$

$$=\lim_{x\to-\infty}\frac{3+\frac{2}{x^2}}{\frac{3}{x^2}-2}$$

$$=\frac{\lim\limits_{x\to-\infty}\left(3+\frac{2}{x^2}\right)}{\lim\limits_{x\to-\infty}\left(\frac{3}{x^2}-2\right)} \qquad \text{LL A5}$$

$$=\frac{\lim\limits_{x\to-\infty}3+\lim\limits_{x\to-\infty}\frac{2}{x^x}}{\lim\limits_{x\to-\infty}\frac{3}{x^2}-\lim\limits_{x\to-\infty}2} \qquad \text{LL A1 and A2}$$

$$=\frac{3+0}{0-2} \qquad \text{LL R2 and R3}$$

$$=-\frac{3}{2}$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{\infty}{\infty}$.

This limit **does** exist.

E2.6.7

$$\lim_{x\to\infty}\sin\left(\frac{\pi e^{3x}}{2e^x+4e^{3x}}\right)=\lim_{x\to\infty}\sin\left(\frac{\pi e^{3x}}{2e^x+4e^{3x}}\cdot\frac{\frac{1}{e^{3x}}}{\frac{1}{e^{3x}}}\right)$$

$$=\lim_{x\to\infty}\sin\left(\frac{\pi}{\frac{2}{e^{2x}}+4}\right)$$

$$=\sin\left(\lim_{x\to\infty}\frac{\pi}{\frac{2}{e^{2x}}+4}\right) \qquad \text{LL A6}$$

$$=\sin\left(\frac{\lim\limits_{x\to\infty}\pi}{\lim\limits_{x\to\infty}\left(\frac{2}{e^{2x}}+4\right)}\right) \qquad \text{LL A5}$$

$$=\sin\left(\frac{\lim\limits_{x\to\infty}\pi}{\lim\limits_{x\to\infty}\frac{2}{e^{2x}}+\lim\limits_{x\to\infty}4}\right) \qquad \text{LL A1}$$

$$=\sin\left(\frac{\pi}{0+4}\right) \qquad \text{LL R2 and R3}$$

$$=1/\sqrt{2}$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{\infty}{\infty}$.

This limit **does** exist.

E2.6.8 $\lim\limits_{x\to\infty} \dfrac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{x}{x}\right)}$ does **not** exist because the domain of the function $y = \dfrac{\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{x}{x}\right)}$ is $\varnothing$.

E2.6.9

$$\begin{aligned}
\lim_{x\to 5}\sqrt{\frac{x^2-12x+35}{5-x}} &= \lim_{x\to 5}\sqrt{\frac{(x-7)(x-5)}{5-x}} \\
&= \lim_{x\to 5}\sqrt{-1\cdot(x-7)} && \text{LL A7} \\
&= \lim_{x\to 5}\sqrt{7-x} \\
&= \sqrt{\lim_{x\to 5}(7-x)} && \text{LL A6} \\
&= \sqrt{\lim_{x\to 5}7-\lim_{x\to 5}x} && \text{LL A2} \\
&= \sqrt{7-5} && \text{LL R1 and R2} \\
&= \sqrt{2}
\end{aligned}$$

We can't apply limit laws A1-A6 yet because the inside limit has the indeterminate form $\frac{0}{0}$.

This limit **does** exist.

E2.6.10

$$\begin{aligned}
\lim_{h\to 0}\frac{4(3+h)^2-5(3+h)-21}{h} &= \lim_{h\to 0}\frac{4\left(9+6h+h^2\right)-15-5h-21}{h} \\
&= \lim_{h\to 0}\frac{36+24h+4h^2-15-5h-21}{h} \\
&= \lim_{h\to 0}\frac{19h+4h^2}{h} \\
&= \lim_{h\to 0}\frac{h(19+4h)}{h} \\
&= \lim_{h\to 0}(19+4h) && \text{LL A7} \\
&= \lim_{h\to 0}19+\lim_{h\to 0}(4h) && \text{LL A1} \\
&= \lim_{h\to 0}19+4\cdot\lim_{h\to 0}h && \text{LL A3} \\
&= 19+4(0) && \text{LL R1 and R2} \\
&= 19
\end{aligned}$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{0}{0}$.

This limit **does** exist.

E2.6.11

$$\begin{aligned}
\lim_{h\to 0}\frac{5h^2+3}{2-3h^2} &= \frac{\lim\limits_{h\to 0}\left(5h^2+3\right)}{\lim\limits_{h\to 0}\left(2-3h^2\right)} && \text{LL A5} \\
&= \frac{\lim\limits_{h\to 0}\left(5h^2\right)+\lim\limits_{h\to 0}3}{\lim\limits_{h\to 0}2-\lim\limits_{h\to 0}\left(3h^2\right)} && \text{LL A1 and A2} \\
&= \frac{5\cdot\lim\limits_{h\to 0}h^2+\lim\limits_{h\to 0}3}{\lim\limits_{h\to 0}2-3\cdot\lim\limits_{h\to 0}h^2} && \text{LL A3} \\
&= \frac{5\left(\lim\limits_{h\to 0}h\right)^2+\lim\limits_{h\to 0}3}{\lim\limits_{h\to 0}2-3\left(\lim\limits_{h\to 0}h\right)^2} && \text{LL A6} \\
&= \frac{5\cdot 0^2+3}{2-3\cdot 0^2} && \text{LL R1 and R2} \\
&= \frac{3}{2}
\end{aligned}$$

This limit **does** exist.

E2.6.12

$$\lim_{h\to 0}\frac{\sqrt{9-h}-3}{h}=\lim_{h\to 0}\left(\frac{\sqrt{9-h}-3}{h}\cdot\frac{\sqrt{9-h}+3}{\sqrt{9-h}+3}\right)$$

$$=\lim_{h\to 0}\frac{(9-h)-9}{h\left(\sqrt{9-h}+3\right)}$$

$$=\lim_{h\to 0}\frac{-h}{h\left(\sqrt{9-h}+3\right)}$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{0}{0}$.

$$=\lim_{h\to 0}\frac{-1}{\left(\sqrt{9-h}+3\right)}$$ LL A7

$$=\frac{\lim_{h\to 0}(-1)}{\lim_{h\to 0}\left(\sqrt{9-h}+3\right)}$$ LL A5

$$=\frac{\lim_{h\to 0}(-1)}{\lim_{h\to 0}\sqrt{9-h}+\lim_{h\to 0}3}$$ LL A1

$$=\frac{\lim_{h\to 0}(-1)}{\sqrt{\lim_{h\to 0}(9-h)}+\lim_{h\to 0}3}$$ LL A6

This limit **does** exist.

$$=\frac{\lim_{h\to 0}(-1)}{\sqrt{\lim_{h\to 0}9-\lim_{h\to 0}h}+\lim_{h\to 0}3}$$ LL A2

$$=\frac{-1}{\sqrt{9-0}+3}$$ LL R1 and R2

$$=-\frac{1}{6}$$

E2.6.13

$$\lim_{\theta\to\frac{\pi}{2}}\frac{\sin\left(\theta+\frac{\pi}{2}\right)}{\sin(2\theta+\pi)}=\lim_{\theta\to\frac{\pi}{2}}\frac{\sin(\theta)\cos\left(\frac{\pi}{2}\right)+\cos(\theta)\sin\left(\frac{\pi}{2}\right)}{\sin(2\theta)\cos(\pi)+\cos(2\theta)\sin(\pi)}$$

$$=\lim_{\theta\to\frac{\pi}{2}}\frac{\cos(\theta)}{-\sin(2\theta)}$$

$$=\lim_{\theta\to\frac{\pi}{2}}\frac{\cos(\theta)}{-2\sin(\theta)\cos(\theta)}$$

We can't apply limit laws A1-A6 yet because the limit has the indeterminate form $\frac{0}{0}$.

$$=\lim_{\theta\to\frac{\pi}{2}}\frac{1}{-2\sin(\theta)}$$ LL A7

$$=\frac{\lim_{\theta\to\frac{\pi}{2}}1}{\lim_{\theta\to\frac{\pi}{2}}\left(-2\sin(\theta)\right)}$$ LL A5

$$=\frac{\lim_{\theta\to\frac{\pi}{2}}1}{-2\lim_{\theta\to\frac{\pi}{2}}\sin(\theta)}$$ LL A3

This limit **does** exist.

$$=\frac{\lim_{\theta\to\frac{\pi}{2}}1}{-2\sin\left(\lim_{\theta\to\frac{\pi}{2}}\theta\right)}$$ LL A6

$$=\frac{1}{-2\sin\left(\frac{\pi}{2}\right)}$$ LL R1 and R2

$$=-\frac{1}{2}$$

E2.6.14 $\lim\limits_{x\to 0^+} \dfrac{\ln\left(x^e\right)}{\ln\left(e^x\right)} = -\infty$ This limit does **not** exist.

Exercise 2.7

	Does limit exist?		Does limit exist?
$\lim\limits_{x\to\infty} e^x = \infty$	No	$\lim\limits_{x\to-\infty} e^x = 0$	Yes
$\lim\limits_{x\to 0} e^x = 1$	Yes	$\lim\limits_{x\to\infty} e^{-x} = 0$	Yes
$\lim\limits_{x\to-\infty} e^{-x} = \infty$	No	$\lim\limits_{x\to 0} e^{-x} = 1$	Yes
$\lim\limits_{x\to\infty} \ln(x) = \infty$	No	$\lim\limits_{x\to 1} \ln(x) = 0$	Yes
$\lim\limits_{x\to 0^+} \ln(x) = -\infty$	No	$\lim\limits_{x\to\infty} \dfrac{1}{x} = 0$	Yes
$\lim\limits_{x\to-\infty} \dfrac{1}{x} = 0$	Yes	$\lim\limits_{x\to 0^+} \dfrac{1}{x} = \infty$	No
$\lim\limits_{x\to 0^-} \dfrac{1}{x} = -\infty$	No	$\lim\limits_{x\to\infty} e^{1/x} = 1$	Yes
$\lim\limits_{x\to\infty} \dfrac{1}{e^x} = 0$	Yes	$\lim\limits_{x\to-\infty} \dfrac{1}{e^x} = \infty$	No
$\lim\limits_{x\to\infty} \dfrac{1}{e^{-x}} = \infty$	No	$\lim\limits_{x\to-\infty} \dfrac{1}{e^{-x}} = 0$	Yes