Supplemental Exercises for the Rates of Change Lab

Exercise 1.1

The function z shown in Figure E1.1 was generated by the formula $y = 2 + 4x - x^2$.

E1.1.1 Simplify the difference quotient for z.

E1.1.2 Use the graph to find the slope of the secant line to z between the points where $x = -1$ and $x = 2$. Check your simplified difference quotient for z by using it to find the slope of the same secant line.

E1.1.3 Replace x with 4 in your difference quotient formula and simplify the result. Then copy Table E1.1 onto your paper and fill in the missing values.

E1.1.4 As the value of h gets closer to 0, the values in the y column of Table 1.1 appear to be converging on a single number; what is this number?

E1.1.5 The value found in problem 1.1.4 is called ***the slope of the tangent line to z at 4***. Draw onto Figure 1 the line that passes through the point $(4, 2)$ with this slope. The line you just drew is called ***the tangent line to z at 4***.

Table E1.1: $y = \dfrac{z(4+h) - z(4)}{h}$

h	y
–0.1	–3.9
–0.01	–3.99
–0.001	–3.999
0.001	–4.001
0.01	–4.01
0.1	–4.01

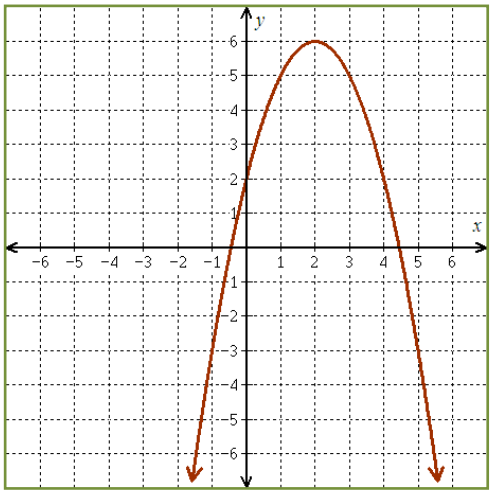

Figure E1.1: $y = 2 + 4x - x^2$

Exercise 1.2

Find the difference quotient for each function showing all relevant steps in an organized manner.

E1.2.1 $f(x) = 3 - 7x$

E1.2.2 $g(x) = \dfrac{7}{x+4}$

E1.2.3 $z(x) = \pi$

E1.2.4 $s(t) = t^3 - t - 9$

E1.2.5 $k(t) = \dfrac{(t-8)^2}{t}$

Exercise 1.3

Suppose that an object is tossed into the air in such a way that the elevation of the object (measured in ft) is given by the function $s(t) = 150 + 60t - 16t^2$ where t is the amount of time that has passed since the object was tossed (measure in seconds).

E1.3.1 Find the difference quotient for s.

E1.3.2 Use the difference quotient to determine the average velocity of the object over the interval $[4, 4.2]$ and then verify the value by calculating $\dfrac{s(4.2) - s(4)}{4.2 - 4}$.

Exercise 1.4

Several applied functions are given below. In each case, find the indicated quantity (including unit) and interpret the value in the context of the application.

E1.4.1 The velocity of a roller coaster (in ft/s) is given by $v(t) = -100\sin\left(\dfrac{\pi t}{15}\right)$ where t is the amount of time (s) that has passed since the coaster topped the first hill. Find and interpret $\dfrac{v(7.5) - v(0)}{7.5\text{s} - 0\text{s}}$.

E1.4.2 The elevation of a ping pong ball relative to the table top (in m) is given by the function $h(t) = 1.1\left|\cos\left(\dfrac{2\pi t}{3}\right)\right|$ where t is the amount of time (s) that has passed since the ball went into play. Find and interpret $\dfrac{h(3) - h(1.5)}{3\text{s} - 1.5\text{s}}$.

E1.4.3 The period of a pendulum (s) is given by $P(x) = \dfrac{6}{x+1}$ where x is the number of swings the pendulum has made. Find and interpret $\dfrac{P(29) - P(1)}{29\text{swing} - 1\text{swing}}$.

E1.4.4 The acceleration of a rocket (mph/s) is given by $h(t) = .02 + .13t$ where t is the amount of time (s) that has passed since lift-off. Find and interpret $\dfrac{h(120) - h(60)}{120\text{s} - 60\text{s}}$.