Supplemental Exercises for the Limits and Continuity Lab

Exercise 2.1

For each of tables E2.2-E2.5, state the limit suggested by the values in the table and state whether or not the limit exists. The correct answer for Table E2.1 has been given to help you understand the directions to the question.

Table E2.1: $y = g(t)$

t	y	Limit	Limit exist?
$-10{,}000$	99.97	$\lim\limits_{t \to -\infty} g(t) = \infty$	No
$-100{,}000$	999.997		
$-1{,}000{,}000$	9999.9997		

Table E2.2: $y = f(x)$

x	y	Limit	Limit exist?
$51{,}000$	-3.2×10^{-5}		
$51{,}0000$	-3.02×10^{-7}		
$5{,}100{,}000$	-3.002×10^{-9}		

Table E2.3: $y = z(t)$

t	y	Limit	Limit exist?
$.33$	$.66$		
$.333$	$.666$		
$.3333$	$.6666$		

Table E2.4: $y = g(\theta)$

θ	y	Limit	Limit exist?
$-.9$	$2{,}999{,}990$		
$-.99$	$2{,}999{,}999$		
$-.999$	$2{,}999{,}999.9$		

Table E2.5: $y = T(t)$

t	y	Limit	Limit exist?
$.778$	$29{,}990$		
$.7778$	$299{,}999$		
$.77778$	$2{,}999{,}999.9$		

Exercise 2.2

Sketch onto Figure E2.1 a function, f, with each of the following properties. Make sure that your graph includes all of the relevant features addressed in lab.

- The only discontinuities on f occur at -4 and 3
- f has no x-intercepts
- f is continuous from the right at -4
- $\lim_{x \to -4^-} f(x) = 1$ and $\lim_{x \to -4^+} f(x) = -2$
- $\lim_{x \to 3} f(x) = -\infty$
- $\lim_{x \to \infty} f(x) = -\infty$
- f has a constant slope of -2 over $(-\infty, -4)$

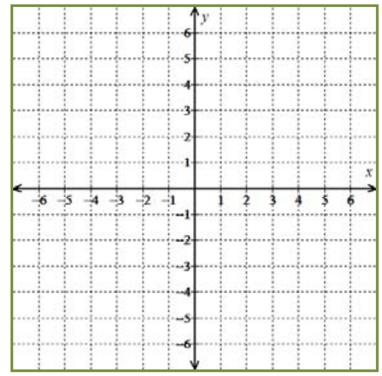

Figure E2.1: f

Exercise 2.3

Sketch onto Figure E2.2 a function, f, that satisfies each of the properties stated below. Assume that there are no intercepts or discontinuities other than those directly implied by the given properties. Make sure that your graph includes all of the relevant features addressed in lab.

- $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 3$
- $\lim_{x \to -4^+} f(x) = -2$ and $\lim_{x \to -4^-} f(x) = 5$
- $\lim_{x \to 3^-} f(x) = -\infty$ and $\lim_{x \to 3^+} f(x) = \infty$
- $f(-2) = 0$, $f(0) = -1$, and $f(-4) = 5$

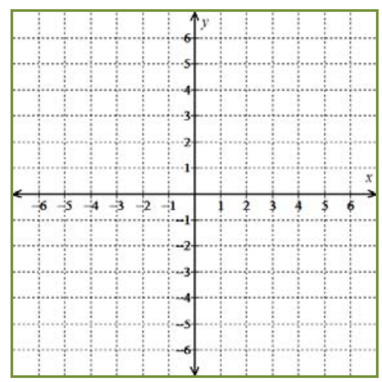

Figure E2.2 f

Exercise 2.4

Determine all of the values of x where the function f (given below) has discontinuities. At each value where f has a discontinuity, determine if f is continuous from either the right or left at x and also state whether or not the discontinuity is removable.

$$f(x) = \begin{cases} \dfrac{2\pi}{x} & \text{if} \quad x \le 3 \\ \sin\left(\dfrac{2\pi}{x}\right) & \text{if} \quad 3 < x < 4 \\ \dfrac{\sin(x)}{\sin(x)} & \text{if} \quad 4 < x \le 7 \\ 3 - \dfrac{2x - 8}{x - 4} & \text{if} \quad x > 7 \end{cases}$$

Exercise 2.5

Determine the value of k that makes the function $g(t)=\begin{cases} t^2+kt-k & \text{if} \quad t\ge 3 \\ t^2-4k & \text{if} \quad t<3 \end{cases}$ continuous over $(-\infty,\infty)$.

Exercise 2.6

Determine the appropriate symbol to write after an equal sign following each of the given limits. In each case, the appropriate symbol is either a real number, ∞, or $-\infty$. Also, state whether or not each limit exists and if the limit exists prove its existence (and value) by applying the appropriate limit laws. The Rational Limit Form table in Appendix C (Pages C3 and C4) summarizes strategies to be employed based upon the initial form of the limit.

E2.6.1 $\lim\limits_{x\to 4^-}\left(5-\dfrac{1}{x-4}\right)$

E2.6.2 $\lim\limits_{x\to\infty}\dfrac{e^{2/x}}{e^{1/x}}$

E2.6.3 $\lim\limits_{x\to 2^+}\dfrac{x^2-4}{x^2+4}$

E2.6.4 $\lim\limits_{x\to 2^+}\dfrac{x^2-4}{x^2-4x+4}$

E2.6.5 $\lim\limits_{x\to\infty}\dfrac{\ln(x)+\ln(x^6)}{7\ln(x^2)}$

E2.6.6 $\lim\limits_{x\to-\infty}\dfrac{3x^3+2x}{3x-2x^3}$

E2.6.7 $\lim\limits_{x\to\infty}\sin\left(\dfrac{\pi e^{3x}}{2e^x+4e^{3x}}\right)$

E2.6.8 $\lim\limits_{x\to\infty}\dfrac{\ln\left(1/x\right)}{\ln\left(x/x\right)}$

E2.6.9 $\lim\limits_{x\to 5}\sqrt{\dfrac{x^2-12x+35}{5-x}}$

E2.6.10 $\lim\limits_{h\to 0}\dfrac{4(3+h)^2-5(3+h)-21}{h}$

E2.6.11 $\lim\limits_{h\to 0}\dfrac{5h^2+3}{2-3h^2}$

E2.6.12 $\lim\limits_{h\to 0}\dfrac{\sqrt{9-h}-3}{h}$

E2.6.13 $\lim\limits_{\theta\to\frac{\pi}{2}}\dfrac{\sin\left(\theta+\frac{\pi}{2}\right)}{\sin(2\theta+\pi)}$

E2.6.14 $\lim\limits_{x\to 0^+}\dfrac{\ln\left(x^e\right)}{\ln\left(e^x\right)}$

Exercise 2.7

Draw sketches of the functions $y = e^x$, $y = e^{-x}$, $y = \ln(x)$, and $y = 1/x$. Note the coordinates of any and all intercepts. This is something you should be able to do from memory/intuition. If you cannot already do so, spend some time reviewing whatever you need to review so that you can do so.

Once you have the graphs drawn, fill in each of the blanks below and also circle whether or not each limit exists. The appropriate symbol for some of the blanks is either ∞ or $-\infty$

		Does limit exist?			Does limit exist?
$\lim\limits_{x\to\infty} e^x =$	______	yes or no	$\lim\limits_{x\to-\infty} e^x =$	______	yes or no
$\lim\limits_{x\to 0} e^x =$	______	yes or no	$\lim\limits_{x\to\infty} e^{-x} =$	______	yes or no
$\lim\limits_{x\to-\infty} e^{-x} =$	______	yes or no	$\lim\limits_{x\to 0} e^{-x} =$	______	yes or no
$\lim\limits_{x\to\infty} \ln(x) =$	______	yes or no	$\lim\limits_{x\to 1} \ln(x) =$	______	yes or no
$\lim\limits_{x\to 0^+} \ln(x) =$	______	yes or no	$\lim\limits_{x\to\infty} \frac{1}{x} =$	______	yes or no
$\lim\limits_{x\to-\infty} \frac{1}{x} =$	______	yes or no	$\lim\limits_{x\to 0^+} \frac{1}{x} =$	______	yes or no
$\lim\limits_{x\to 0^-} \frac{1}{x} =$	______	yes or no	$\lim\limits_{x\to\infty} e^{1/x} =$	______	yes or no
$\lim\limits_{x\to\infty} \frac{1}{e^x} =$	______	yes or no	$\lim\limits_{x\to-\infty} \frac{1}{e^x} =$	______	yes or no
$\lim\limits_{x\to\infty} \frac{1}{e^{-x}} =$	______	yes or no	$\lim\limits_{x\to-\infty} \frac{1}{e^{-x}} =$	______	yes or no