

Supplemental Exercises for the Introduction to the First Derivative Lab

Exercise 3.1

Find the first derivative formula for each of the following functions twice: first by evaluating

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ and then by evaluating } \lim_{t \rightarrow x} \frac{f(t) - f(x)}{t - x}.$$

E3.1.1 $f(x) = x^2$

E3.1.2 $f(x) = \sqrt{x}$

E3.1.3 $f(x) = 7$

Exercise 3.2

It can be shown that $\lim_{h \rightarrow 0} \frac{\sin(h)}{h} = 1$ and $\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} = 0$. Use these limits to help you to establish the first derivative formula for $\sin(x)$.

Hint: Begin with $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h}$ and use the sum formula for $\sin(x+h)$.

Exercise 3.3

Suppose that an object is tossed into the air in such a way that the elevation of the object (measured in ft) is given by the function $s(t) = 150 + 60t - 16t^2$ where t is the amount of time that has passed since the object was tossed (measure in seconds).

E3.3.1 Find the velocity function for this motion and use that function to determine the velocity of the object 4.1 s into its motion.

E3.3.2 Find the acceleration function for this motion and use that function to determine the acceleration of the object 4.1 s into its motion.

Exercise 3.4

Determine the unit on the first derivative function for each of the following functions. Remember, *we do not simplify derivative units in any way, shape, or form.*

E3.4.1 $R(p)$ is Carl's heart rate (beats/min) when he jogs at a rate of p (measured in ft/min).

E3.4.2 $F(v)$ is the fuel consumption rate (gal/mi) of Hanh's pick-up when she drives it on level ground at a constant speed of v (measured in mi/hr).

E3.4.3 $v(t)$ is the velocity of the space shuttle (mi/hr) where t is the amount of time that has passed since lift-off (measured in seconds).

E3.4.4 $h(t)$ is the elevation of the space shuttle (mi) where t is the amount of time that has passed since lift-off (measured in seconds).

Exercise 3.5

Referring to the functions in Exercise 3.4, write sentences that explain the meaning of each of the following function values.

E3.5.1 $R(300) = 84$

E3.5.2 $R'(300) = .02$

E3.5.3 $F(50) = .03$

E3.5.4 $F'(50) = -.0006$

E3.5.5 $v(20) = 266$

E3.5.6 $v'(20) = 18.9$

E3.5.7 $h(20) = .7$

E3.5.8 $h'(20) = .074$

Exercise 3.6

It can be shown that the derivative formula for the function $f(x) = \ln(x) + 2x$ is

$$f'(x) = \frac{2x + 1}{x}.$$

E3.6.1 Determine the equation of the tangent line to f at 1.

E3.6.2 A graph of f is shown in Figure E3.1; axis scales have deliberately been omitted from the graph. The graph shows that f quickly resembles a line. In a detailed sketch of f we would reflect this apparent linear behavior by adding a skew asymptote. What is the slope of this skew asymptote?

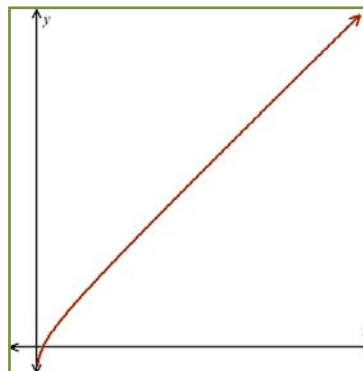


Figure E3.1: $f(x) = \ln(x) + 2x$