Supplemental Exercises for the Implicit Differentiation Lab

Exercise 7.1

The curve $x\sin(xy)=y$ is shown in Figure E7.1. Find a formula for $\frac{dy}{dx}$ and use that formula to determine the x-coordinate at each of the two points the curve crosses the x-axis. (Note: The tangent line to the curve is vertical at each of these points.) Scales have deliberately been omitted in Figure E7.1.

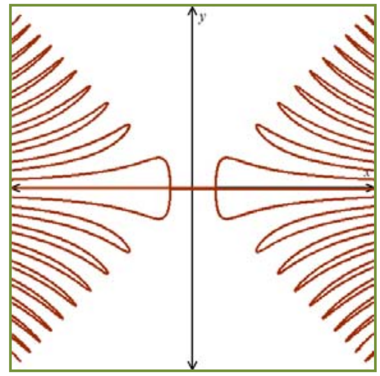

Figure E7.1: $x\sin(xy)=y$

Exercise 7.2

Solutions to the equation $\ln\left(x^2y^2\right)=x+y$ are graphed in Figure E7.2. Determine the equation of the tangent line to this curve at the point $(1,-1)$.

Hint: It is easier to differentiate if you first use rules of logarithms to completely expand the logarithmic expression.

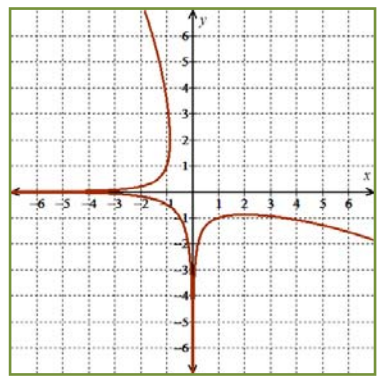

Figure E7.2: $\ln\left(x^2y^2\right)=x+y$

Exercise 7.3

You have formulas that allow you to differentiate x^2, 2^x, and 2^2. You don't, however, have a formula to differentiate x^x. In this exercise you are going to use a process called **logarithmic differentiation** to determine the derivative formula for the function $y=x^x$. Example E7.1 (page A22) shows this process for a different function.

The function $y=x^x$ is only defined for positive values of x (which in turn means y is also positive), so we can say that $\ln(y)=\ln\left(x^x\right)$. What you need to do is use implicit differentiation to find a formula for $\frac{dy}{dx}$ ***after first applying the power rule of logarithms to the logarithmic expression on the right side of the equal sign***. Once you have your formula for $\frac{dy}{dx}$, substitute x^x for y. Voila! You will have the derivative formula for x^x. So go ahead and do it.

Exercise 7.4

In the olden days (pre-symbolic calculators) we would use the process of logarithmic differentiation to find derivative formulas for complicated functions. The reason this process is "simpler" than straight forward differentiation is that we can obviate the need for the product and quotient rules if we completely expand the logarithmic expression before taking the derivative.

Use the process of logarithmic differentiation to find a first derivative formula for each of the following functions. The process of logarithmic differentiation is illustrated in .

7.4.1 $y=\dfrac{x\sin(x)}{\sqrt{x-1}}$ **7.4.2** $y=\dfrac{e^{2x}}{\sin^4(x)\sqrt[4]{x^5}}$ **7.4.3** $y=\dfrac{\ln(4x^3)}{x^5\ln(x)}$

Example E7.1

$$y=\frac{x^{e^x}}{4x+1}$$

$$\ln(y)=\ln\left(\frac{x^{e^x}}{4x+1}\right)$$

Set the natural logarithms of the two expressions equal to one another.

$$\ln(y)=\ln\left(x^{e^x}\right)-\ln(4x+1)$$

$$\ln(y)=e^x\ln(x)-\ln(4x+1)$$

Completely expand the logarithmic expression on the right side of the equal sign.

$$\frac{d}{dx}(\ln(y))=\frac{d}{dx}\left(e^x\ln(x)-\ln(4x+1)\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}\left(e^x\right)\cdot\ln(x)+e^x\cdot\frac{d}{dx}(\ln(x))-\frac{1}{4x+1}\cdot\frac{d}{dx}(4x+1)$$

$$\frac{1}{y}\frac{dy}{dx}=e^x\ln(x)+\frac{e^x}{x}-\frac{4}{4x+1}$$

$$\frac{dy}{dx}=y\cdot\left(e^x\ln(x)+\frac{e^x}{x}-\frac{4}{4x+1}\right)$$

Solve for $\dfrac{dy}{dx}$ after going through the process of implicit differentiation.

$$\frac{dy}{dx}=\frac{x^{e^x}}{4x+1}\cdot\left(e^x\ln(x)+\frac{e^x}{x}-\frac{4}{4x+1}\right)$$

Replace y with its original formula in the formula for $\dfrac{dy}{dx}$.