Supplemental Exercises for the Functions, Derivatives, and Antiderivatives Lab

Exercise 4.1

Sketch the first derivatives of the functions shown in figures E4.1a, E4.2a, and E4.3a onto, respectively, figures E4.1b, E4.2b, and E4.3b.

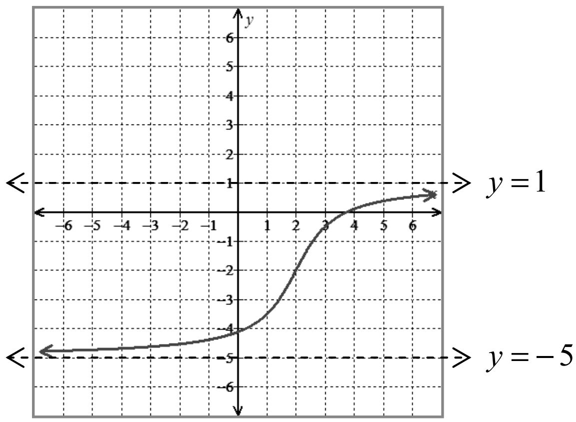

Figure E4.1a

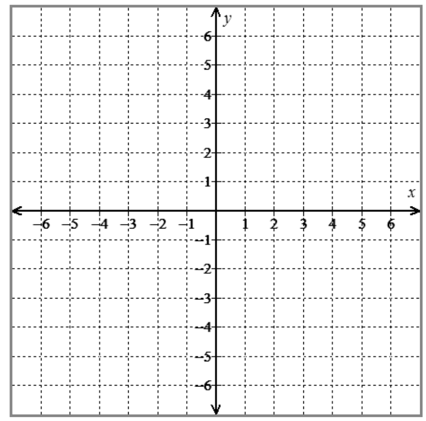

Figure E4.1b

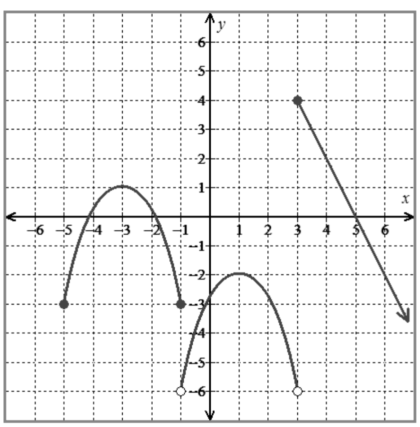

Figure E4.2a

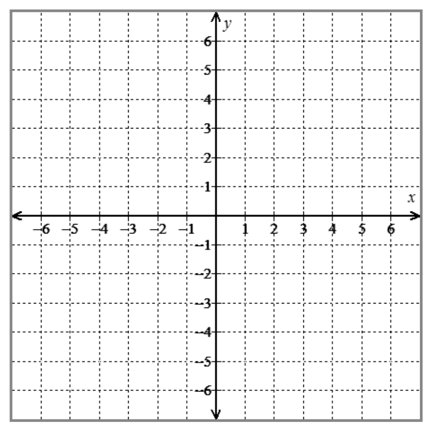

Figure E4.2b

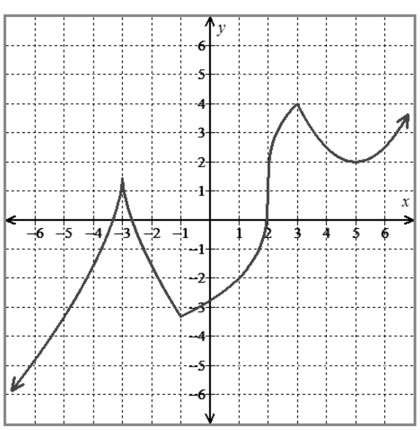

Figure E4.3c

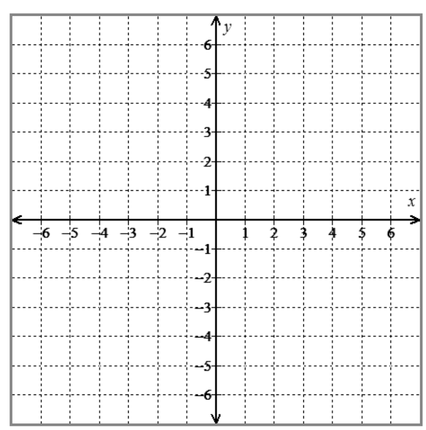

Figure E4.3b

Exercise 4.2

At the bottom of the page seven curves are identified as Figures A-G. For each statement below, identify the figure letters that go along with the given statement. Assume each statement is being made only in reference to the portion of the curves shown in each of the figures. Assume in all cases that the axes have the traditional positive/negative orientation. The first question has been answered for you to help you get started.

<u>C, D, and E</u> **E4.2.1** Each of these functions is always increasing.

________ **E4.2.2** The first derivative of each of these functions is always increasing.

________ **E4.2.3** The second derivative of each of these functions is always negative.

________ **E4.2.4** The first derivative of each of these functions is always positive.

________ **E4.2.5** Any antiderivative of each of these functions is always concave up.

________ **E4.2.6** The second derivative of each of these functions is always equal to zero.

________ **E4.2.7** Any antiderivative of each of these functions graphs to a line.

________ **E4.2.8** The first derivative of each of these functions is a constant function.

________ **E4.2.9** The second derivative of each of these functions is never positive.

________ **E4.2.10** The first derivative of each of these functions **must** be linear.

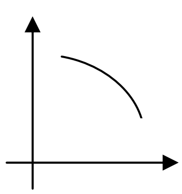

Figure A

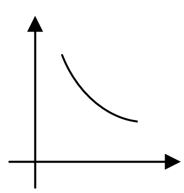

Figure B

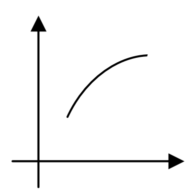

Figure C

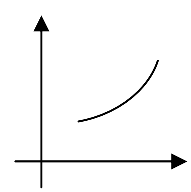

Figure D

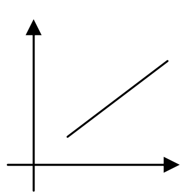

Figure E

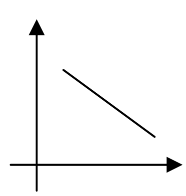

Figure F

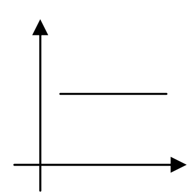

Figure G

Exercise 4.3

As water drains from a tank the drainage rate continually decreases over time. Suppose that a tank that initially holds 360 gal of water drains in exactly 6 minutes. Answer each of the following questions about this problem situation.

E4.3.1 Suppose that $V(t)$ is the amount of water (gal) left in the tank where t is the amount of time that has elapsed (s) since the drainage began. What are the units on $V'(45)$ and $V''(45)$? Which of the following is the most realistic value for $V'(45)$?

a 0.8 **b** 1.2 **c** -0.8 **d** -1.2

E4.3.2 Suppose that $R(t)$ is the water's flow rate (gal/s) where t is the amount of time that has elapsed (s) since the drainage began. What are the units on $R'(45)$ and $R''(45)$? Which of the following is the most realistic value for $R'(45)$?

a 1.0 **b** 0.001 **c** -1.0 **d** -0.001

E4.3.3 Suppose that $V(t)$ is the amount of water (gal) left in the tank where t is the amount of time that has elapsed (s) since the drainage began. Which of the following is the most realistic value for $V''(45)$?

a 1.0 **b** 0.001 **c** -1.0 **d** -0.001

Exercise 4.4

For each statement (E4.4.1-E4.4.5), decide which of the following is true.

i. The statement is true regardless of the specific function f.
ii. The statement is true for some functions and false for other functions.
iii. The statement is false regardless of the specific function f.

E4.4.1 If a function f is increasing over the entire interval $(-3,7)$, then $f'(0)>0$.

E4.4.2 If a function f is increasing over the entire interval $(-3,7)$, then $f'(4)=0$.

E4.4.3 If a function f is concave down over the entire interval $(-3,7)$, then $f'(-2)<0$.

E4.4.4 If the slope of f is increasing over the entire interval $(-3,7)$, then $f'(0)>0$.

E4.4.5 If a function f has a local maximum at 3, then $f'(3)=0$.

Exercise 4.5

For a certain function g, $g'(t) = -8$ at all values of t on $(-\infty, -3)$ and $g'(t) = -2$ at all values of t on $(-3, \infty)$.

E4.5.1 Janice thinks that g must be discontinuous at -3 but Lisa disagrees. Who's right?

E4.5.2 Lisa thinks that the graph of g'' is the line $y = 0$ but Janice disagrees. Who's right?

Exercise 4.6

Each statement below is in reference to the function shown in Figure E4.4. Decide whether each statement is true or false. A caption has been omitted from Figure E4.4 because the identity of the curve changes from question to question.

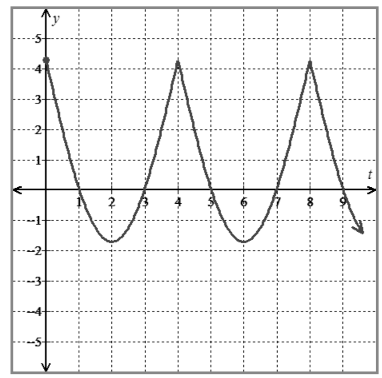

Figure E4.4

E4.6.1 If the given function is f', then $f(3) > f(2)$.

E4.6.2 If the given function is f, then $f'(3) > f'(2)$.

E4.6.3 If the given function is f, then $f'(1) > f''(1)$.

E4.6.4 If the given function is f', then the tangent line to f at 6 is horizontal.

E4.6.5 If the given function is f', then f'' is always increasing.

E4.6.6 If the given function is f', then f is nondifferentiable at 4.

E4.6.7 If the given function is f', then f' is nondifferentiable at 4.

E4.6.8 The first derivative of the given function is periodic (starting at 0).

E4.6.9 Antiderivatives of the given function are periodic (starting at 0).

E4.6.10 If the given function was measuring the rate at which the volume of air in your lungs was changing t seconds after you were frightened, then there was the same amount of air in your lungs 9 seconds after the fright as was there 7 seconds after the fright.

E4.6.11 If the given function is measuring the position of a weight attached to a spring relative to a table edge (with positive and negative positions corresponding, respectively, to the weight being above and below the edge of the table), then the weight is in the same position 9 seconds after it begins to bob as it is 7 seconds after the bobbing commenced (assuming that t represents the number of seconds that pass after the weight begins to bob).

E4.6.12 If the given function is measuring the velocity of a weight attached to a spring relative to a table edge (with positive and negative positions corresponding, respectively, to the weight being above and below the edge of the table), then the spring was moving downward over the interval $(1,3)$ (assuming that t represents the number of seconds that pass after the weight begins to bob).

Exercise 4.7

Complete as many cells in Table E4.1 as possible so that when read left to right the relationships between the derivatives, function, and antiderivative are always true. Please note that several cells will remain blank.

Table E4.1

f''	f'	f	F
		Positive	
		Negative	
		Constantly Zero	
		Increasing	
		Decreasing	
		Constant	
		Concave Up	
		Concave Down	
		Linear	

Exercise 4.8

Answer each question about the function f whose first derivative is shown in Figure E4.5. The correct answer to one or more of the questions may be "there's no way of knowing."

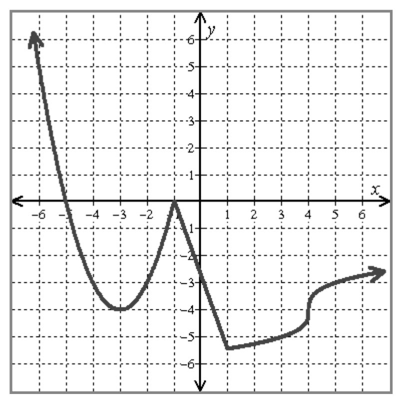

Figure E4.5: f'

E4.8.1 Where, over $(-6,6)$, is f' nondifferentiable?

E4.8.2 Where, over $(-6,6)$, are antiderivatives of f nondifferentiable?

E4.8.3 Where, over $(-6,6)$, is f decreasing?

E4.8.4 Where, over $(-6,6)$, is f concave down?

E4.8.5 Where, over $(-6,6)$, is f'' decreasing?

E4.8.6 Where, over $(-6,6)$, is f'' positive?

E4.8.7 Where, over $(-6,6)$, are antiderivatives of f concave up?

E4.8.8 Where, over $(-6,6)$, are antiderivatives of f increasing?

E4.8.9 Where, over $(-6,6)$, does f have its maximum value?

E4.8.10 Suppose that $f(-3)=14$. What, then, is the equation of the tangent line to f at -3?

Exercise 4.9

A certain antiderivative, F, of the function f shown in Figure E4.6a passes through the points $(-3,6)$ and $(3,-2)$. Draw this antiderivative (over the interval $(-6,6)$) onto Figure E4.6b.

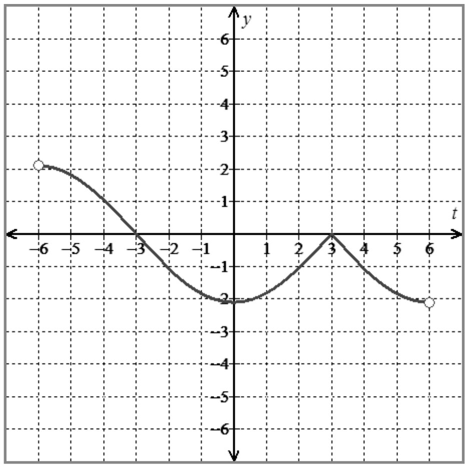

Figure E4.6a: f

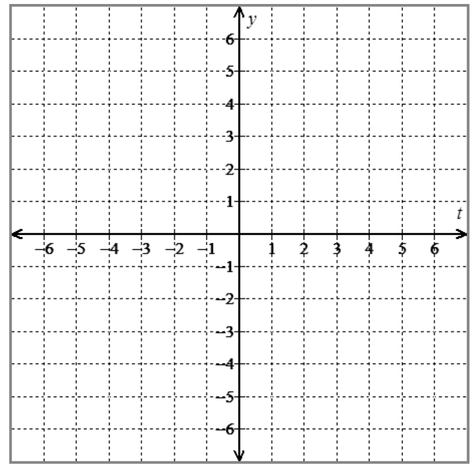

Figure E4.6b: F

Exercise 4.10

Draw onto Figure E4.7 the function f given the following properties of f.

- $f(0)=f(4)=3$
- The only discontinuity on f occurs at the vertical asymptote $x=2$.
- $f'(-1)=0$
- $f'(x)>0$ on $(-\infty,-1)$, $(-1,2)$, and $(4,\infty)$
- $f'(x)<0$ on $(2,4)$
- $f''(x)>0$ on $(-1,2)$ and $(2,4)$
- $f''(x)<0$ on $(-\infty,-1)$
- $f''(x)=0$ on $(4,\infty)$
- $\lim_{x\to 4^-} f'(x)=-1$ and $\lim_{x\to 4^+} f'(x)=1$

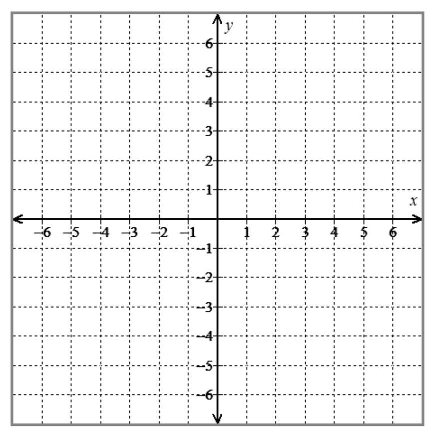

Figure E4.7 f