Supplemental Exercises for the Derivative Formulas Lab

Exercise 5.1

Apply the constant factor and power rules of differentiation to find the first derivative of each of the following functions. In each case take the derivative with respect to the independent variable suggested by the expression on the right side of the equal sign. Make sure that you assign the proper name to the derivative function. Please note that there is no need to employ either the quotient rule or the product rule to find any of these derivative formulas.

E5.1.1 $f(x)=\dfrac{\sqrt[11]{x^6}}{6}$ **E5.1.2** $y=-\dfrac{5}{t^7}$ **E5.1.3** $y(u)=12\sqrt[3]{u}$

E5.1.4 $z(\alpha)=e\alpha^{\pi}$ **E5.1.5** $z=\dfrac{8}{\sqrt{t^7}}$ **E5.1.6** $T=\dfrac{4\sqrt[3]{t^7}}{t^2}$

Exercise 5.2

Apply the constant factor and product rules of differentiation to find the first derivative of each of the following functions. Write out the Leibniz notation for the product rule as applied to the given function. In each case take the derivative with respect to the independent variable suggested by the expression on the right side of the equal sign. Make sure that you assign the proper name to the derivative function.

E5.2.1 $y=5\sin(x)\cos(x)$ **E5.2.2** $y=\dfrac{5t^2 e^t}{7}$ **E5.2.3** $F(x)=4x\ln(x)$

E5.2.4 $z=x^2\sin^{-1}(x)$ **E5.2.5** $T(t)=\left(1+t^2\right)\tan^{-1}(t)$ **E5.2.6** $T=\dfrac{x^7\cdot 7^x}{3}$

Exercise 5.3

Apply the constant factor and quotient rules of differentiation to find the first derivative of each of the following functions. Write out the Leibniz notation for the quotient rule as applied to the given function. In each case take the derivative with respect to the independent variable suggested by the expression on the right side of the equal sign. Make sure that you assign the proper name to the derivative function.

E5.3.1 $q(\theta)=\dfrac{4e^{\theta}}{e^{\theta}+1}$ **E5.3.2** $u(x)=2\dfrac{\ln(x)}{x^4}$ **E5.3.3** $F=\dfrac{\sqrt{t}}{3t^2-5\sqrt{t^3}}$

E5.3.4 $F(x)=\dfrac{\tan(x)}{\tan^{-1}(x)}$ **E5.3.5** $p=\dfrac{\tan^{-1}(t)}{1+t^2}$ **E5.3.6** $y=\dfrac{4}{\sin(\beta)-2\cos(\beta)}$

Exercise 5.4

Multiple applications of the product rule are required when finding the derivative formula of the product of three of more functions. After simplification, however, the predictable pattern shown below emerges.

$$\frac{d}{dx}(f\,g) = \frac{d}{dx}(f)\,g + f\,\frac{d}{dx}(g)$$

$$\frac{d}{dx}(f\,g\,h) = \frac{d}{dx}(f)\,g\,h + f\,\frac{d}{dx}(g)\,h + f\,g\,\frac{d}{dx}(h)$$

$$\frac{d}{dx}(f\,g\,h\,k) = \frac{d}{dx}(f)\,g\,h\,k + f\,\frac{d}{dx}(g)\,h\,k + f\,g\,\frac{d}{dx}(h)\,k + f\,g\,h\,\frac{d}{dx}(k)$$

E5.4.1 Apply the product rule twice to the expression $\frac{d}{dx}\big(f(x)\,g(x)\,h(x)\big)$ to verify the formula stated above for the product of three functions. The first application of the product rule is shown below to help you get you started.

$$\frac{d}{dx}\big(f(x)\,g(x)\,h(x)\big) = \frac{d}{dx}\big(f(x)\big)\big[g(x)\,h(x)\big] + f(x)\,\frac{d}{dx}\big[g(x)\,h(x)\big]$$

E5.4.2 Apply the formula shown for the product of four functions to determine the derivative with respect to x of the function $f(x) = x^2\,e^x\,\sin(x)\cos(x)$

Exercise 5.5

Find the first derivative with respect to x for each of the following functions and for each function find the equation of the tangent line at the stated value of x. Make sure that you use appropriate techniques of differentiation.

E5.5.1 $f(x) = \dfrac{x^3 + x^2}{x}$; tangent line at 5

E5.5.2 $h(x) = \dfrac{x}{1+x}$; tangent line at -2

E5.5.3 $K(x) = \dfrac{1+x}{2x+2}$; tangent line at 8

E5.5.4 $r(x) = 3\sqrt[3]{x}\,e^x$; tangent line at 0

Exercise 5.6

Find the second derivative of y with respect to t if $y = 4\sqrt{t}\,t^5$. Make sure that you use appropriate techniques of differentiation and that you use proper notation on both sides of the equal sign.

Exercise 5.7

A function f is shown in Figure E5.1. Each question below is in reference to this function. Please note that the numeric endpoints of each interval answer are all integers.

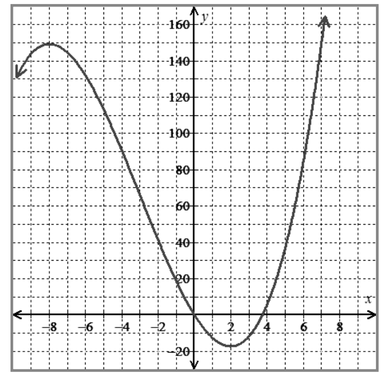

Figure E5.1

E5.7.1 Over what intervals is f' positive and over what intervals is f' negative?

E5.7.2 Over what intervals is f' increasing and over what intervals is f' decreasing?

E5.7.3 Over what intervals is f'' positive and over what intervals is f'' negative?

E5.7.4 The formula for f is $f(x)=\dfrac{x^3}{3}+3x^2-16x$. Find the formulas for f' and f'', and then graph the derivatives and verify your answers to parts a-c.

Exercise 5.8

A function g is shown in Figure E5.2. Each question below is in reference to this function.

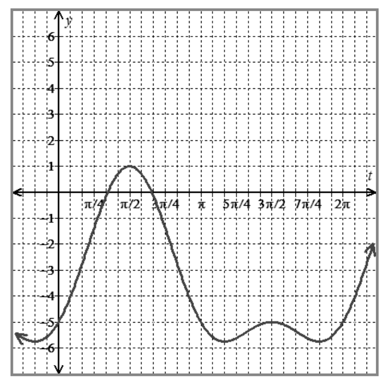

Figure E5.2

E5.8.1 Where over the interval $[0,2\pi]$ does $g'(t)=0$? Please note that the t-scale in Figure E5.2 is $\dfrac{\pi}{12}$.

E5.8.2 The formula for g is $g(t)=3\sin(t)+3\sin^2(t)-5$. Find the formula for g' and use that formula to solve the equation $g'(t)=0$ over the interval $[0,2\pi]$. Compare your answers to those found in part a.

Hint: $\sin^2(t)=\sin(t)\sin(t)$

Exercise 5.9

Find the equation of the tangent line to f' at π if $f(x)=\sin(2x)$.

Hint: Begin by applying a double angle identity. Also, make sure that you read the question carefully.

Exercise 5.10

In Table E5.1 several function and derivative values are given for the functions f and g. This entire problem is based upon these two functions.

Suppose that you were asked to find the value of $h'(2)$ where $h(x)=f(x)g(x)$. The first thing you would need to do is find a formula for $h'(x)$. You would then replace x with 2, substitute the appropriate function and derivative values, and simplify. This is illustrated in example E5.1.

Following the process outlined in example E5.1, find each of the following.

E5.10.1 Find $h'(4)$ where $h(x)=f(x)g(x)$.

E5.10.2 Find $h''(2)$ where $h(x)=f(x)g(x)$.

E5.10.3 Find $k'(3)$ where $k(x)=\dfrac{g(x)}{f(x)}$.

E5.10.4 $p'(4)$ where $p(x)=6\sqrt{x}\,f(x)$.

E5.10.5 $r'(1)$ where $r(x)=\left[g(x)\right]^2$.

E5.10.6 $s'(2)$ where $s(x)=x\,f(x)\,g(x)$.

E5.10.7 $F'(4)$ where $F(x)=\sqrt{x}\,g(4)$.

E5.10.8 $T''(0)$ where $T(x)=\dfrac{f(x)}{e^x}$.

Example E5.1

$$\begin{aligned} h'(x) &= \frac{d}{dx}\left(f(x)\right)g(x)+f(x)\frac{d}{dx}\left(g(x)\right) \\ &= f'(x)g(x)+f(x)g'(x) \end{aligned}$$

$$\begin{aligned} h'(2) &= f'(2)g(2)+f(2)g'(2) \\ &= (7)(11)+(4)(37) \\ &= 225 \end{aligned}$$

Table E5.1

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
0	2	−1	−2	−3	−3	−2
1	1	0	4	−5	2	16
2	4	7	10	11	37	58
3	15	20	16	87	126	124
4	46	39	22	289	293	214