Supplemental Exercises for the Critical Numbers and Graphing from Formulas Lab

Exercise 9.1

The sine and cosine function are called circular functions because for any given value of t the point $\left(\cos(t), \sin(t)\right)$ lies on the circle with equation $x^2 + y^2 = 1$.

There are analogous functions called hyperbolic sine and hyperbolic cosine. As you might suspect, these functions generate points that lie on a hyperbola; specifically, for all values of t the point $\left(\cosh(t), \sinh(t)\right)$ lies on the hyperbola $x^2 - y^2 = 1$. It turns out that there are alternate formulas for the hyperbolic functions. Specifically:

$$\cosh(t) = \frac{e^t + e^{-t}}{2} \text{ and } \sinh(t) = \frac{e^t - e^{-t}}{2}$$

E9.1.1 Use the exponential formulas to verify the identity $\cosh^2(t) - \sinh^2(t) = 1$.

E9.1.2 Use the exponential formulas to determine the first derivatives (with respect to t) of $\cosh(t)$ and $\sinh(t)$. Determine, by inference, the second derivative formulas for each of these functions.

E9.1.3 Determine the critical numbers of $\cosh(t)$ and $\sinh(t)$.

E9.1.4 Determine the intervals over which $\cosh(t)$ and $\sinh(t)$ are increasing/decreasing and over which they are concave up/concave down.

E9.1.5 Use the exponential formulas to determine each of the following limits: $\lim\limits_{t \to -\infty} \cosh(t)$, $\lim\limits_{t \to \infty} \cosh(t)$, $\lim\limits_{t \to -\infty} \sinh(t)$, and $\lim\limits_{t \to \infty} \sinh(t)$.

E9.1.6 Use the information you determined in problems E9.1.3-E9.1.5 to help you draw freehand sketches of $y = \cosh(t)$ and $y = \sinh(t)$.

E9.1.7 There are four more hyperbolic functions that correspond to the four additional circular functions; e.g. $\tanh(t) = \dfrac{\sinh(t)}{\cosh(t)}$. Find the exponential formulas for these four functions.

E9.1.8 What would you guess to be the first derivative of $\tanh(t)$? Take the derivative of the exponential formula for $\tanh(t)$ to verify your suspicion.

E9.1.9 Let $f(t) = \tanh(t)$. What is the sign on $f'(t)$ at all values of t? What does this tell you about the function f? What are the values of $f(0)$ and $f'(0)$?

E9.1.10 Use the exponential formula to determine $\lim\limits_{t \to -\infty} \tanh(t)$ and $\lim\limits_{t \to \infty} \tanh(t)$.

E9.1.11 Use the information you determined in problems E9.1.9-E9.1.10 to help you draw a freehand sketch of $y = \tanh(t)$.

Exercise 9.2

Consider the function $k(t) = t^{8/3} - 256t^{2/3}$.

E9.2.1 What are the critical numbers of k? Remember to show all relevant work! Remember that your formula for $k'(t)$ needs to be a single, completely factored, fraction!

E9.2.2 Create an increasing/decreasing table for k.

E9.2.3 State each local minimum point and local maximum point on k.

Exercise 9.3

Consider the function $f(x) = \cos^2(x) + \sin(x)$ over the restricted domain $[0, 2\pi]$.

E9.3.1 What are the critical numbers of f? Remember to show all relevant work! Remember that your formula for $f'(x)$ needs to be a single, completely factored, fraction!

E9.3.2 Create an increasing/decreasing table for f.

E9.3.3 State each local minimum point and local maximum point on f.

Exercise 9.4

Consider $g(t) = \dfrac{t+9}{t^3}$. Find (and completely simplify) $g''(t)$ and state all numbers where $g''(t)$ is zero or undefined. Then construct a concavity table for g and state all of the inflection points on g.

Exercise 9.5

For each of the following functions build increasing/decreasing tables and concavity tables and then state all local minimum points, local maximum points, and inflection points on the function. Also determine and state all horizontal asymptotes and vertical asymptotes for the function. Finally, draw a detailed sketch of the function.

E9.5.1 $f(x) = \dfrac{x-3}{(x+2)^2}$ **E9.5.2** $g(x) = x^{2/3}(x+5)$ **E9.5.3** $k(x) = \dfrac{(x-4)^2}{x+3}$