Supplemental Exercises for the Chain Rule Lab

Exercise 6.1

The functions $f(x)=e^{3x}$, $g(x)=\left(e^{x}\right)^{3}$, and $h(x)=\left(e^{3}\right)^{x}$ are equivalent. Find the first derivative formulas for each of the functions (without altering their given forms) and then explicitly establish that the derivative formulas are the same.

Exercise 6.2

Consider the function $k(\theta)=\sin^{-1}(\sin(\theta))$.

E6.2.1 Over the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $k(\theta)=\theta$. What does this tell you about the formula for $k'(\theta)$ over $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$?

E6.2.2 Use the chain rule and an appropriate trigonometric identity to verify your answer to question E6.2.1. Please note that $\cos(\theta)>0$ over $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

E6.2.3 What is the constant value of $k'(\theta)$ over $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$. Hint: What is the sign on $\cos(\theta)$ over that interval?

E6.2.4 What is the value of $k'\left(\frac{\pi}{2}\right)$?

Exercise 6.3

Consider the function f shown in Figure E6.1.

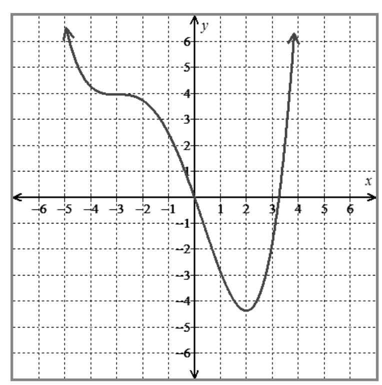

Figure E6.1: f

E6.3.1 Suppose that $g(x)=\left[f(x)\right]^{4}$. Over what intervals is g' positive?

E6.3.2 Suppose that $r(x)=e^{f(x)}$. Over what intervals is r' positive?

E6.3.3 Suppose that $w(x)=e^{f(-x)}$. Over what intervals is w' positive?

E6.3.4 Suppose that $h(x)=\dfrac{1}{f(x)}$. Is h nondifferentiable at -3?

(Please note that f has a horizontal tangent line at -3.)

Exercise 6.4

Decide whether or not it is necessary to use the chain rule when finding the derivative with respect to x of each of the following functions.

E6.4.1 $f(x)=\ln(x+1)$ **E6.4.2** $f(x)=\dfrac{2}{x^5}$ **E6.4.3** $f(x)=\cos(\pi)$

E6.4.4 $f(x)=\cos(x)$ **E6.4.5** $f(x)=\cos(\pi x)$ **E6.4.6** $f(x)=\dfrac{\cos(\pi x)}{\pi}$

Exercise 6.5

Find the first derivative with respect to x of each of the following functions. In all cases, look for appropriate simplifications before taking the derivative. Please note that some of the functions will be simpler to differentiate if you first use the rules of logarithms to expand and simplify the logarithmic expression.

E6.5.1 $f(x)=\tan^{-1}\left(\sqrt{x}\right)$

E6.5.2 $f(x)=e^{e^{\sin(x)}}$

E6.5.3 $f(x)=\sin^{-1}(\cos(x))$

E6.5.4 $f(x)=\tan(x\sec(x))$

E6.5.5 $f(x)=\tan(x)\sec(\sec(x))$

E6.5.6 $f(x)=\sqrt[3]{\left(\sin\left(x^2\right)\right)^2}$

E6.5.7 $f(x)=4x\sin^2(x)$

E6.5.8 $f(x)=\ln(x\ln(x))$

E6.5.9 $f(x)=\ln\left(\dfrac{5}{xe^x}\right)$

E6.5.10 $f(x)=2\ln\left(\sqrt[3]{x\tan^2(x)}\right)$

E6.5.11 $f(x)=\ln\left(\dfrac{e^{x+2}}{\sqrt{x+2}}\right)$

E6.5.12 $f(x)=\ln\left(x^e+e\right)$

E6.5.13 $f(x)=\sec^4\left(e^x\right)$

E6.5.14 $f(x)=\sec^{-1}\left(e^x\right)$

E6.5.15 $f(x)=\csc\left(\dfrac{1}{\sqrt{x}}\right)$

E6.5.16 $f(x)=\dfrac{1}{\csc\left(\sqrt{x}\right)}$

E6.5.17 $f(x)=\dfrac{\tan^{-1}(2x)}{2}$

E6.5.18 $f(x)=x^3\sin\left(\dfrac{x}{3}\right)$

E6.5.19 $f(x)=\dfrac{4}{\sqrt{\dfrac{3}{x^7}}}$

E6.5.20 $f(x)=\dfrac{e^{xe^x}}{x}$

E6.5.21 $f(x)=xe^{xe^2}$

E6.5.22 $f(x)=\dfrac{\sin^5(x)-\sqrt{\sin(x)}}{\sin(x)}$

E6.5.23 $f(x)=4x\sin(x)\cos\left(x^2\right)$

E6.5.24 $f(x)=\sin\left(x\cos^2(x)\right)$