Rational Limit Forms

Form	Example	Course of Action
$\dfrac{\text{real number}}{\text{non-zero real number}}$	$\lim_{x\to 3}\dfrac{2x+16}{5x-4}$ (the form is $\dfrac{22}{11}$)	The value of the limit is the number to which the limit form simplifies; for example, $\lim_{x\to 3}\dfrac{2x+16}{5x-4}=2$. You can immediately begin applying limit laws if you are "proving" the limit value.
$\dfrac{\text{non-zero real number}}{\text{zero}}$	$\lim_{x\to 7^-}\dfrac{x+7}{7-x}$ (the form is $\dfrac{14}{0}$)	The limit value doesn't exist. The expression whose limit is being found is either increasing without bound or decreasing without bound (or possibly both if you have a two-sided limit). You may be able to communicate the non-existence of the limit using ∞ or $-\infty$. For example, if the value of x is a little less than 7, the value of $\dfrac{x+7}{7-x}$ is positive. Hence you could write $\lim_{x\to 7^-}\dfrac{x+7}{7-x}=\infty$.
$\dfrac{\text{zero}}{\text{zero}}$	$\lim_{x\to -2}\dfrac{x^2-4}{x^2+3x+2}$ (the form is $\dfrac{0}{0}$)	This is an ***indeterminate form limit***. You do not know the value of the limit (or even if it exists) nor can you begin to apply limit laws. You need to manipulate the expression whose limit is being found until the resultant limit no longer has indeterminate form. For example: $\lim_{x\to -2}\dfrac{x^2-4}{x^2+3x+2}=\lim_{x\to -2}\dfrac{(x-2)(x+2)}{(x+1)(x+2)}$ $=\lim_{x\to -2}\dfrac{(x-2)}{(x+1)}$ (The limit form is now $\dfrac{-4}{-1}$; you may begin applying the limit laws.)
$\dfrac{\text{real number}}{\text{infinity}}$	$\lim_{x\to 0^+}\dfrac{1-4e^x}{\ln(x)}$ (the form is $\dfrac{-3}{-\infty}$)	The limit value is zero and this is justified by logic similar to that used to justify Limit Law R3.

Rational Limit Forms

Form	Example	Course of Action
$\dfrac{\text{infinity}}{\text{real number}}$	$\lim\limits_{x\to 0^+}\dfrac{\ln(x)}{1-4e^x}$ (the form is $\dfrac{-\infty}{-3}$)	The limit value doesn't exist. The expression whose limit is being found is either increasing without bound or decreasing without bound (or possibly both if you have a two-sided limit). You may be able to communicate the non-existence of the limit using ∞ or $-\infty$. For example, you can write $\lim\limits_{x\to 0^+}\dfrac{\ln(x)}{1-4e^x}=\infty$.
$\dfrac{\text{infinity}}{\text{infinity}}$	$\lim\limits_{x\to\infty}\dfrac{3+e^x}{1+3e^x}$	This is an ***indeterminate form limit***. You do not know the value of the limit (or even if it exists) nor can you begin to apply limit laws. You need to manipulate the expression whose limit is being found until the resultant limit no longer has indeterminate form. For example: $\lim\limits_{x\to\infty}\dfrac{3+e^x}{1+3e^x}=\lim\limits_{x\to\infty}\left(\dfrac{3+e^x}{1+3e^x}\cdot\dfrac{1/e^x}{1/e^x}\right)=\lim\limits_{x\to\infty}\dfrac{\frac{3}{e^x}+1}{\frac{1}{e^x}+3}$ (The limit form is now $\frac{1}{3}$; you may begin applying the limit laws.)

There are other indeterminate limit forms, although the two mentioned in Table 1 are the only two *rational* indeterminate forms. The other indeterminate forms are discussed in MTH 252.