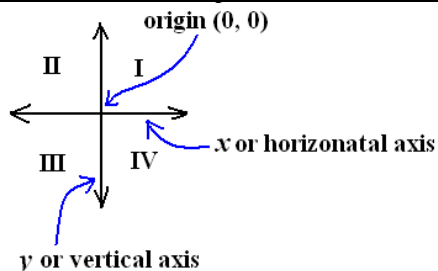


# Things to know for graphing lines:

- A **SOLUTION** to a linear equation/ graph are the corresponding values [ordered pairs] that make the statement true.
- Quadrants: In the Cartesian coordinate system the plane can be separated into 4 sections or **QUADRANTS**: We often refer to the horizontal axis as the  $x$ -axis and the vertical axis as the  $y$ -axis because our points are often of the form  $(x, y)$  BUT this is not always true! What really counts, is that in the ordered pair the first value is the horizontal component and the second value is the vertical component (horizontal, vertical)!



**NOTE:** *Intercepts are always points!  
We do not know where to graph 5 on the plane!  
We do know where to graph the point  $(5, 0)$  on the plane!  
So when saying a point such as the  $x$ -intercept  $(5, 0)$  we say it is "the point five comma zero".  
Points in 2 dimensions are always ordered pairs!*

- Intercepts: **INTERCEPTS** are where the graph intersects or touches the axis. For the curve to touch the vertical-axis (usually the  $y$ -axis) there is no horizontal movement so the ordered pair must be of the form  $(0, y)$ . For the curve to touch the horizontal-axis (usually the  $x$ -axis) there is no horizontal movement so the ordered pair must be of the form  $(x, 0)$ .

In other words, the  $y$ -intercept always means  $x = 0$  and the  $x$ -intercept means that  $y = 0$

$$y - \text{int} \Rightarrow x = 0$$

$$x - \text{int} \Rightarrow y = 0$$

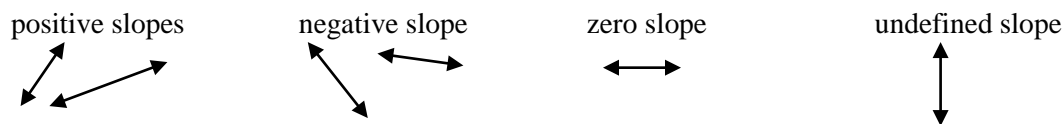
- To uniquely determine a line we **NEED**: 2 points or the slope and a point
- The **SLOPE** ( $m$ ) is a **ratio** or often a **rate of change**:

$$\text{slope} = m = \frac{\text{the change vertically}}{\text{the change horizontally}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

given points  $(x_1, y_1)$  and  $(x_2, y_2)$

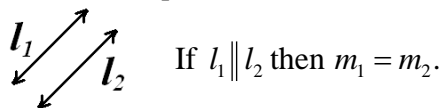
**Grade** is the slope as a percent [so 'run' is always 100].  $\text{grade} = m \text{ as a } \% = \frac{\text{rise}}{100}$

**Pitch** is the absolute value of a slope ( $m \geq 0$ ) often used in construction and the 'run' is usually 12.

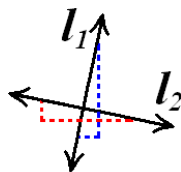


**If you see parallel or perpendicular THINK SLOPE!**

**Parallel lines** have the same slope: (think railroad tracks)



**Perpendicular lines** have slopes that are negative reciprocals:



Notice: If a line has a positive slope then the perpendicular has a negative slope.

Also: The rise of a line looks like the run of the perpendicular and vice versa.

So... If  $l_1 \perp l_2$  then  $m_1 = \frac{-1}{m_2}$  or  $m_1 \cdot m_2 = -1$

- The **SLOPE-INTERCEPT FORM** of a line is:  $y = mx + b$   
 Where  $m = \text{slope}$  & the y-intercept is  $(0, b)$

- The **POINT-SLOPE FORM** of a line is:  $y - y_1 = m(x - x_1)$   
 Given point  $(x_1, y_1)$  and slope  $= m$

*Note:* We never leave an equation of a line in this form. We always leave equations of lines in the Standard form or slope-intercept form. The Point-slope form is only used to find the equation of a line.

To find the equation of a line given two points:

- Find the slope using the slope formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Plug the slope,  $m$ , and one of your points (no, it does not matter) into the *point-slope form*.
- Algebraically convert this into the desired *Standard form* or *slope-intercept form*.

- The **STANDARD FORM** of a line is:  $Ax + By = C$   
 Where  $A, B,$  &  $C$  are integers and  $A \geq 0$  [ $A$  is a whole number].

### Why the standard form?

**Optional (not sure why)... but really, really useful!** *This sums up the whole term of algebra (algebraic manipulation and graphing). Students often ask why is the standard form so important? The wording is from some math that is further ahead, but this form is in some ways the **most** useful. If we use the ideas of solving for a variable and that the intercepts always have one coordinate of zero, we can show what the intercepts and the slope are **in general**. If we do this for variables - and variables just represent numbers - then we have shown this is **true for ANY number** (where defined)! Then looking at this the standard form we get ways to quickly find BOTH intercepts and the slope! (Algebra rocks!) We can find the intercepts because we know that for the vertical or y-intercept  $x$  is always 0 and for the horizontal or x-intercept  $y$  is always 0. Also we know that the slope-intercept form gives us the slope so if we convert the standard form into the slope-intercept form we see what the slope is. Algebra rocks! And this for  $m$  becomes the most useful! ☺*

$x\text{-int} \Rightarrow y = 0$ $Ax + B(0) = C$ $Ax = C$ $\frac{Ax}{A} = \frac{C}{A}$ $x = \frac{C}{A}$	$y\text{-int} \Rightarrow x = 0$ $A(0) + By = C$ $By = C$ $\frac{By}{B} = \frac{C}{B}$ $y = \frac{C}{B}$	$Ax + By = C$ $-Ax \quad -Ax$ $By = -Ax + C$ $\frac{By}{B} = \frac{-Ax + C}{B}$ $y = -\frac{A}{B}x + \frac{C}{B}$
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So... the  $x$ -intercept is  $\left(\frac{C}{A}, 0\right)$       Then the  $y$ -intercept is  $\left(0, \frac{C}{B}\right)$       and the slope  $= m = -\frac{A}{B}$

**Notice this would be much harder (and thus less useful) if  $A, B,$  &  $C$  were not integers!**

For example to graph the equation  $2x - 3y = 12$  using what we just discovered, we have:

$$y\text{-intercept of } \left(0, \frac{12}{-3}\right) = (0, -4)$$

$$x\text{-intercept of } \left(\frac{12}{2}, 0\right) = (6, 0)$$

$$\text{and slope } = m = -\frac{2}{-3} = \frac{2}{3}$$

**BEHOLD THE POWER OF ALGEBRA!!!**

## THE FOLLOWING IS OPTIONAL.... But very useful! ☺

We actually don't need the point-slope form at all...

Using the slope-intercept form of  $y = mx + b$  if we know a point  $(x_1, y_1)$  and slope =  $m$  and we plug this into the slope-intercept form then the only unknown is  $b$ . Given an equation with one unknown we can solve it. Then (*and here is the key*) we can plug that back into the slope-intercept form along with our slope and have the equation of the line in slope-intercept form!

For example, given the point  $(3, 5)$  and a slope =  $\frac{4}{7}$  we plug the  $y$ , its corresponding  $x$  and the slope into

$$y = mx + b \text{ and solve for } b: \quad 5 = \frac{4}{7}(3) + b$$

$$5 = \frac{12}{7} + b$$

$$5 - \frac{12}{7} = \frac{12}{7} + b - \frac{12}{7}$$

$$\frac{5\left(\frac{7}{7}\right) - 12}{1\left(\frac{7}{7}\right) - 7} = b$$

$$\frac{35}{7} - \frac{12}{7} = b$$

$$\frac{23}{7} = b$$

We are not yet done as we have to plug our  $m = \frac{4}{7}$  and  $b = \frac{23}{7}$  back into the point-slope form and we get

$$y = \frac{4}{7}x + \frac{23}{7} .$$

If we then wanted to get this into standard form we have to get rid of the fraction and get the term with an  $x$  in it on the other side. As shown to the right

$$y = \frac{4}{7}x + \frac{23}{7}$$

$$-\frac{4}{7}x + y = \frac{4}{7}x + \frac{23}{7} - \frac{4}{7}x$$

$$-7\left(-\frac{4}{7}x + y\right) = -7\left(\frac{23}{7}\right)$$

$$-7\left(-\frac{4}{7}x\right) + (-7)(y) = -23$$

$$4x - 7y = -23$$

So we see that given the point  $(3, 5)$  and a slope =  $\frac{4}{7}$

we have an equation in slope-intercept form of  $y = \frac{4}{7}x + \frac{23}{7}$

and in standard form of  $4x - 7y = -23$ .

From the standard form we see that we have a:  $y$ -intercept of  $\left(0, \frac{C}{B}\right) = \left(0, \frac{-23}{-7}\right) = \left(0, \frac{23}{7}\right)$

$x$ -intercept of  $\left(\frac{C}{A}, 0\right) = \left(\frac{-23}{4}, 0\right)$

and slope =  $m = -\frac{A}{B} = -\frac{4}{-7} = \frac{4}{7}$

**BEHOLD THE POWER OF ALGEBRA!!!** ☺