1. a. $f'(x) = -\frac{20}{x^6}$ b. $f'(x) = \frac{7}{4}x^{3/4}$

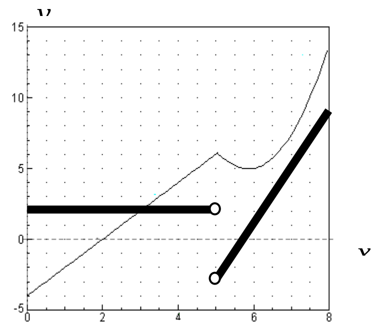
2. a. $\frac{dy}{dx} = 6x\cos\left(3x^2\right)$ b. $\frac{dy}{dx} = 24x^7 e^x + 3x^8 e^x$ c. $\frac{dy}{dx} = \frac{3}{x} + \frac{x}{x^2+2} - \frac{1}{2}\cot(x)$

d. $\frac{dy}{dx} = \frac{14x}{\left(x^2+4\right)^2}$ e. $\frac{dy}{dx} = \frac{1}{x}$ f. $\frac{dy}{dx} = 2\left(3x^2+1\right)\sin\left(x^3+x\right)\cos\left(x^3+x\right)$

g. $\frac{dy}{dx} = \frac{d}{dx}(\ln(e^{3x^2+5x-4})) = \frac{d}{dx}(3x^2+5x-4) = 6x+5$ h. $\frac{dy}{dx} = \frac{e^{x+y}+1}{2y-e^{x+y}}$ i. $\frac{dy}{dx} = \frac{ye^{xy}}{1-xe^{xy}}$

3. $y = -\frac{4}{43}x - \frac{35}{43}$ 4. about 0.0053 cm/min 5. $x = \frac{5}{3}$ 6. 6

7.



8. a. $f'(x) = 2x\,g(x) + x^2 g'(x)$

b. $f'(x) = 2g(x)\,g'(x)$

c. $f'(x) = e^x g'\left(e^x\right)$

d. $f'(x) = \frac{\left(4g(x)+4xg'(x)\right)\left(x^3-2\right)-12x^3 g(x)}{\left(x^3-2\right)^2}$

9. 12 ft/sec

10.

-3 $\frac{3}{7}$ 5 x

Sign on $f'(x)$	Positive	Positive	Negative	Positive
Behavior of $f(x)$	Increasing	Increasing	Decreasing	Increasing

Figure 1: $f'(x) = (x-5)^3(x+3)^2(7x-3)$

11. $g''(x) = \frac{14x-20}{25(x-1)^{8/5}}$. The only point of inflection on $y = g(x)$ occurs at $x = \frac{10}{7}$.

12. $f(2)$ must exist, $\lim_{x\to 2} f(x)$ must exist, and $\lim_{x\to 2} f(x)$ must equal $f(2)$

13. a. about 20.67 cm b. ball moving towards paddle ($v(4)$ negative so $p(t)$ decreasing)
c. speeding up ($v(4)$ and $a(4)$ have same sign - accelerating force acting with motion)

14. a. The particle moves right when the velocity is positive. This occurs on $[0,3)$.
The particle moves left when the velocity is negative. This occurs on $(3, 10]$.

b. On the time interval $[0,3)$ the object moved right $\frac{1}{6}$ cm: $s(3)-s(0)=\frac{1}{6}$.

On the time interval $(3, 10]$ the object moved left $\frac{49}{654}$ cm: $s(10)-s(3)=-\frac{49}{654}$.

So the total distance traveled by the particle was $\frac{1}{6}$ cm + $\frac{49}{654}$ cm = $\frac{79}{327}$ cm.

c. The speed increases when the velocity and the acceleration have the same sign. Since the denominator of each function is always positive, we can focus our attention on the numerators of the functions. Table 1 summarizes the result.

Table 1: Answer for 14(c)

Interval	$v(t)$	$a(t)$	Conclusion
$(0,3)$	pos	neg	slowing down
$(3,\sqrt{27})$	neg	neg	speeding up
$(\sqrt{27},10)$	neg	pos	slowing down

15.

$$\lim_{t\to 1}\frac{t-1}{t^3-t}=\lim_{t\to 1}\frac{t-1}{t(t+1)(t-1)}$$
$$=\lim_{t\to 1}\frac{1}{t(t+1)}$$
$$=\frac{1}{1(1+1)}$$
$$=\frac{1}{2}$$

16.

$$\lim_{t\to\infty}\frac{1+t-3t^2}{4-2t+5t^2}=\lim_{t\to\infty}\frac{\frac{1}{t^2}+\frac{t}{t^2}-\frac{3t^2}{t^2}}{\frac{4}{t^2}-\frac{2t}{t^2}+\frac{5t^2}{t^2}}$$
$$=\lim_{t\to\infty}\frac{\frac{1}{t^2}+\frac{1}{t}-3}{\frac{4}{t^2}-\frac{2}{t}+5}$$
$$=-\frac{3}{5}$$

17. $V'(5)$ is the rate at which the volume of juice remaining in the glass was changing 5 seconds after Jimbo began to drink. Since the volume was decreasing, the rate is clearly negative. Specifically, since the volume decreased at a constant rate and decreased by 1/2 liter in 10 seconds, we have:

$$\text{V}'(5)=\frac{-1/2\text{ liter}}{10\text{sec}}=-\frac{1}{20}\frac{\text{liter}}{\text{sec}}$$

18. The area of the ripple is increasing at a rate of 24π cm^2/sec at the instant when the ripple has a radius of 10 cm.

19. a. $h'(4)=-4.5$ b. $s'(4)=2$ c. $\frac{d}{dx}\left(f\left(f(4)\right)\right)=0$

20. a. The critical number of $g(x)$ is $x=-1$. [Note: $x=-2$ is not a critical number since it is not in the domain of $g(x)$; $g(x)$ cannot have a local extreme value at a point where it has no value.]

b. Table 2: $g'(x)=\dfrac{(x+1)e^x}{(x+2)^2}$

Interval	Sign of $g'(x)$	Graphical behavior of $g(x)$
(-∞, -2)	Negative	Decreasing
(-2, -1)	Negative	Decreasing
(-1, 2)	Positive	Increasing
(2,∞)	Positive	Increasing

c. $g(x)$ has a local minimum value of $\dfrac{1}{e}$ which occurs at $x=-1$.

d. $g(x)$ has no local maximum values.

e. Table 3: $g''(x)=\dfrac{(x^2+2x+2)e^x}{(x+2)^3}$

Interval	Sign of $g''(x)$	Graphical behavior of $g(x)$
(-∞, -2)	Negative	Concave down
(-2, 2)	Positive	Concave up
(2, ∞)	Positive	Concave up

f. g has no inflection points since the only place g changes concavity is at $x=-2$ which is not in the domain of g.

g. g has a horizontal asymptote at $y=0$ since $\lim_{x\to-\infty}\dfrac{(x-2)e^x}{x^2-4}=0$.

h. g has a hole at $\left(2,\dfrac{e^2}{4}\right)$ since $\lim_{x\to2}\dfrac{(x-2)e^x}{x^2-4}=\dfrac{e^2}{4}$.

i. g has a vertical asymptote at $x=-2$ since $\lim_{x\to-2^-}\dfrac{(x-2)e^x}{x^2-4}=-\infty$ and $\lim_{x\to-2^+}\dfrac{(x-2)e^x}{x^2-4}=\infty$

j.

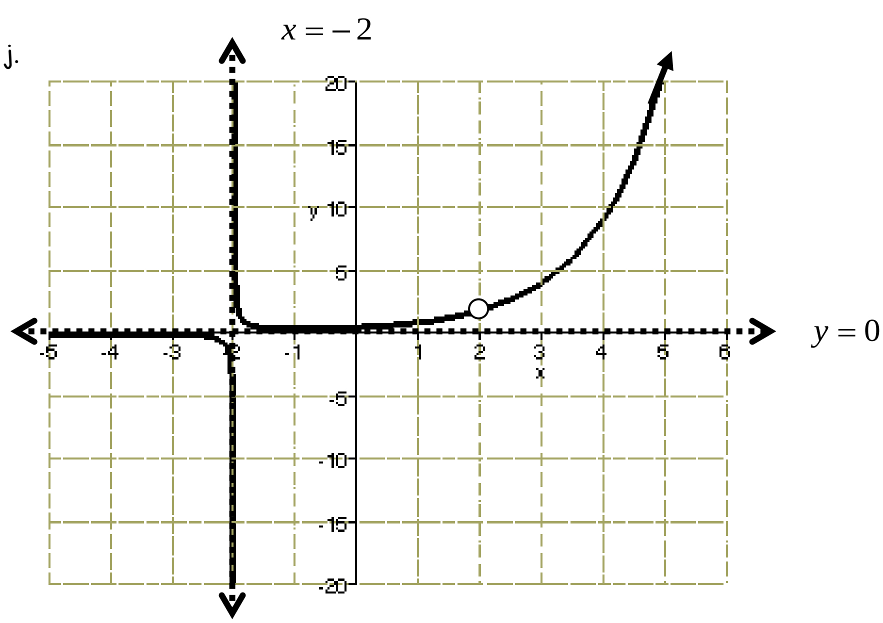


Figure 2 $g(x)=\dfrac{(x-2)e^x}{x^2-4}$

21. $h(x)$ has a local minimum at $(0,0)$ and a local maximum at $\left(-2,\frac{4}{e^2}\right)$.

22. The critical numbers of $m(x)$ are $x=0$ and $x=\frac{1}{4}$. $m(x)$ increases over the interval $\left(\frac{1}{4},\infty\right)$ and decreases over the interval $\left(-\infty,\frac{1}{4}\right)$. $m(x)$ is concave up on the intervals $\left(-\infty,-\frac{1}{2}\right)$ and $(0,\infty)$; $m(x)$ is concave down on the interval $\left(-\frac{1}{2},0\right)$.

23. $g'(\theta)=2\sin(\theta)\cos(\theta)(\cos(\theta)+\sin(\theta))(\cos(\theta)-\sin(\theta))$. The critical numbers for $g(\theta)=\sin^2(\theta)\cos^2(\theta)$ over $(0,2\pi)$ are:

$$\theta=\frac{\pi}{4},\theta=\frac{\pi}{2},\theta=\frac{3\pi}{4},\theta=\pi,\ \theta=\frac{5\pi}{4},\theta=\frac{3\pi}{2},\theta=\frac{7\pi}{4}.$$

24. The only critical number of $g(x)$ is $x=e^{3/2}$.

25. $\lim_{x\to 3^+}\frac{7}{3-x}=-\infty$

26. $\lim_{x\to\infty}\frac{1}{\ln(x)}=0$

27. $\lim_{x\to 5^-}\frac{2}{5-x}=\infty$

28. $\lim_{x\to\infty}\frac{4x^2+1}{4x^3-1}=0$