

MTH 251 Final Exam Review Questions

1. Find $f'(x)$ for each of the following functions.

a. $f(x) = \frac{4}{x^5}$ b. $f(x) = \sqrt[4]{x^7}$

2. Find $\frac{dy}{dx}$ for each of the following functions.

a. $y = \sin(3x^2)$

b. $y = 3x^8 e^x$

c. $y = \ln\left(x^3 \sqrt{\frac{x^2 + 2}{\sin x}}\right)$

d. $y = \frac{x^2 - 3}{x^2 + 4}$

e. $y = \pi^2 + \ln x$

f. $y = \sin^2(x^3 + x)$

g. $y = \ln(e^{3x^2 + 5x - 4})$

h. $e^{x+y} + x - y^2 = 3$

i. $y = e^{-xy}$

3. Find the equation of the tangent line to the curve $x^3 - y^4 + 2x^2 y^5 = y$ at the point $(2, -1)$.

4. If a snowball melts so that its surface area decreases at a rate of $2 \text{ cm}^2 / \text{min}$, find the rate at which the radius decreases at the instant the radius is 15 cm.

5. Find the value of x where the function $f(x) = 3x^2 - 6x + 2$ has a tangent line parallel to the line given by the equation $12x - 3y = 14$. Write your solution with complete sentences which orient the reader to the purpose of all equations and formulas written in your problem solving process.

6. Find $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$. Write all of the mathematics using proper notation and formatting. Show all non-trivial algebraic steps.

No calculators will be allowed on the final exam!

MTH 251 Final Exam Review Questions

7. Figure 1 is a sketch of the function $y=f(x)$. Sketch onto the same axes system the function $y=f'(x)$.

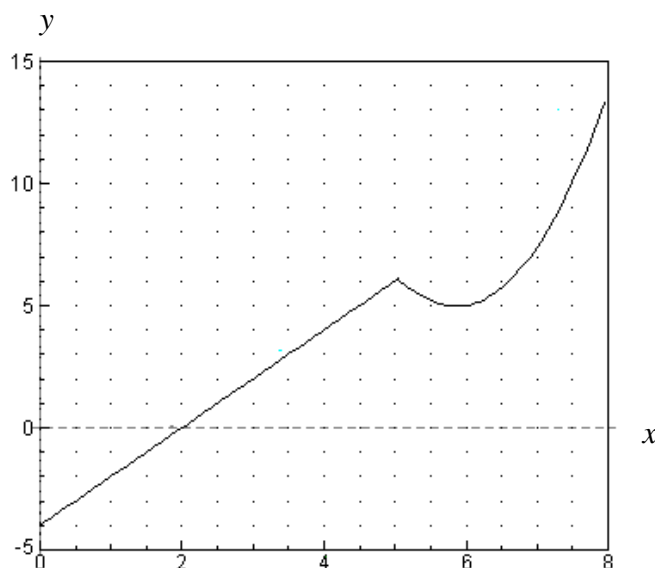


Figure 1: $y=f(x)$

8. In each of the expressions below, a function $f(x)$ has been defined in terms of the function $g(x)$ combined with some other function. For each expression, find the corresponding expression for $f'(x)$.
- a. $f(x) = x^2 g(x)$ b. $f(x) = [g(x)]^2$ c. $f(x) = g(e^x)$ d. $f(x) = \frac{4xg(x)}{x^3 - 2}$
9. Greg is 6 ft tall and walks with a speed of 8 ft/s away from a street light that is atop an 18-ft pole. How fast is the tip of his shadow moving along the ground when he is 100 ft from the light pole?
10. Build an increasing/decreasing number line for the function $f(x) = (x - 5)^4(x + 3)^3$. Show all relevant work in an organized manner.
11. Find the values of x where the function $g(x) = x(x - 1)^{2/5}$ has inflection point(s). Show all relevant work in an organized manner.
12. What three things must be true about the function $y = f(x)$ at $x = 2$ if the function is continuous at the point where $x = 2$?

MTH 251 Final Exam Review Questions

13. JimBob was hard at play with his paddleball. t seconds after JimBob began his play, the position of the ball relative to the paddle (in cm) was given by the function:

$$P(t) = 12.5 - 12.5 \cos t$$

(A position of zero means the ball was touching the paddle; positive positions mean the ball was away from the paddle; this position function is never negative.)

Answer each of the following questions about JimBob's paddleball extravaganza.

(5 points each)

- How far was the ball from the paddle at $t = 4$? **How do you know?**
 - Was the ball moving towards or away from the paddle at $t = 4$? **How do you know?**
 - Was the motion of the ball speeding up or slowing down at $t = 4$? **How do you know?**
14. A particle moves along the x -axis for ten seconds. The position function for the particle, $s(t)$, is given below.

Given: $s(t) = \frac{t}{t^2 + 9}$, $s'(t) = \frac{9 - t^2}{(t^2 + 9)^2}$, $s''(t) = \frac{2t(t^2 - 27)}{(t^2 + 9)^3}$ all for $0 \leq t \leq 10$.

- When was the particle moving right? When was the particle moving left? How do you know?
 - What was the total distance covered by the particle over the ten seconds? How do you know?
 - Over what time interval(s) was the speed of the particle's motion increasing? How do you know?
15. Find $\lim_{t \rightarrow 1} \frac{t-1}{t^3-t}$ algebraically.
16. Find $\lim_{t \rightarrow \infty} \frac{1+t-3t^2}{4-2t+5t^2}$ algebraically.
17. Jimbo is drinking juice again. Jimbo is drinking the juice through a straw and is sucking on the straw in such a way that the volume of juice remaining in the glass is decreasing at a constant rate. Let $y = V(t)$ be the function that gives the volume of juice remaining in the glass t seconds after Jimbo begins his suction action; $V(t)$ is expressed in liters. It takes exactly 10 seconds to drain the glass. There is $1/2$ liter of juice in the glass when Jimbo begins to drink. What is the value of $V'(5)$ (including units)? Explain!
18. Peggy Sue accidentally drops an earring in the toilet bowl. (Ewww! ☹) A circular ripple forms whose radius increases at a constant rate of 1.2 cm/sec. How fast is the area of the ripple increasing at the instant when the ripple has a radius of 10 cm?

19. Consider the functions f and g shown in Table 2.

a. Find $h'(4)$ given that

$$h(x) = f(x) \cdot g(x).$$

b. Find $s'(4)$ given that

$$s(x) = f(g(x))$$

c. Find $\frac{d}{dx}(f(f(4)))$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	6	-4	-2	3
1	4.2	-1.5	-1.5	2
2	3.7	0	0	1
3	3.8	0.5	0.5	0
4	4.5	0	1	-1
5	5.6	-1.5	1.2	-2
6	7	-4	1.6	-3
7	8.2	-7.5	1	-4
8	8.7	-12	0	-5

20. Suppose that you are given the following information about a function and its derivatives.

$$g(x) = \frac{(x-2)e^x}{x^2-4} \qquad g'(x) = \frac{(x+1)e^x}{(x+2)^2} \qquad g''(x) = \frac{(x^2+2x+2)e^x}{(x+2)^3}$$

Answer each of the questions below, using **calculus** to justify your answers.

- What are the critical numbers of g ?
- Create a table which clearly shows the interval(s) on which g is increasing and/or decreasing.
- What are the local minimum values of g and where do they occur?
- What are the local maximum values of g and where do they occur?
- Create a table which clearly shows the interval(s) on which g is concave up and/or concave down.
- Does g have any inflection points? If so, what are they?
- Does g have any horizontal asymptotes? If so, what are they?
- Does g have any holes? If so, what are the coordinates?
- Does g have any vertical asymptotes? If so, what are they?
- Using the information found in parts a - i, sketch a graph of g . Be sure to label all important features. The graph should be of "master-piece" quality as opposed to a haphazard doodle.

21. Find the exact coordinates of the local extrema (maximum or minimum) for the function $h(x) = x^2 e^x$ using the First Derivative Test.

22. Find the critical numbers of $m(x) = x^{4/3} - x^{1/3}$. Where is $m(x)$ increasing? decreasing? concave up? concave down?

23. Find the critical numbers of $g(\theta) = \sin^2(\theta)\cos^2(\theta)$. over $(0, 2\pi)$

24. Find the critical numbers of $g(x) = \frac{1 - \ln(x)}{x^2}$.

MTH 251 Final Exam Review Questions

25. Find $\lim_{x \rightarrow 3^+} \frac{7}{3-x}$.

26. Find $\lim_{x \rightarrow \infty} \frac{1}{\ln(x)}$.

27. Find $\lim_{x \rightarrow 5^-} \frac{2}{5-x}$.

28. Find $\lim_{x \rightarrow \infty} \frac{4x^2 + 1}{4x^3 - 1}$.