

Section 5.2 Three Exponent Rules

Write $3^2 \cdot 3^5$ in expanded form. How many factors of 3 are there in this product?

$$3^2 \cdot 3^5 = (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$$

There are 7 factors of 3.

The Product Rule

$$b^m \cdot b^n = b^{m+n}$$

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

Use the product rule.

$$9^3 \cdot 9^4 = 9^7$$

$$x^2 \cdot x^7 = x^9$$

$$y \cdot y^4 \cdot y^6 = y^{1+4+6}$$

$$x \cdot x^4 \cdot y^3 = x^5 y^3$$

Write $(3^4)^2$ in expanded form. How many factors of 3 are there in this product?

$$\begin{aligned} (3^4)^2 &= 3^4 \cdot 3^4 \\ &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \end{aligned}$$

There are 8 factors of 3.

The Power Rule (Powers to Powers)

$$(b^m)^n = b^{mn}$$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

Use the power rule

$$(7^3)^4 = 7^{12}$$

$$(x^4)^5 = x^{20}$$

Use the power rule

$$\begin{aligned} [(-2)^4]^6 &= (-2)^{24} \\ &= 2^{24} \end{aligned}$$

A negative number multiplied by itself an even number of times is positive.

Write $(3x)^4$ in expanded form. How many factors of 3 are there in this product? How many factors of x are in this product?

$$(3x)^4 = 3x \cdot 3x \cdot 3x \cdot 3x$$

There are 4 factors of 3 and 4 factors of x .

Product to Powers

$$(ab)^n = a^n b^n$$

When a product is raised to a power, raise each factor to the power.

Use the product to powers rule.

$$\begin{aligned} (4a)^3 &= 4^3 a^3 \\ &= 64a^3 \end{aligned}$$

$$\begin{aligned} (-2xy^3)^4 &= (-2)^4 x^4 (y^3)^4 \\ &= 16x^4 y^{12} \end{aligned}$$

$$\begin{aligned} (3x)^4 &= 3^4 x^4 \\ &= 81x^4 \end{aligned}$$

$$\begin{array}{r} 27 \\ \times 3 \\ \hline 81 \end{array}$$

Section 2.1 The Addition Property of Equality

Recall that an equation is a statement that two algebraic expressions are equal. **Solving an equation** is the process of finding the number (or numbers) that make the equation a true statement. These numbers are called the **solutions**, or **roots**, of the equation, and we say that they **satisfy** the equation.

Definition of a Linear Equation in One Variable

A linear equation in one variable, x , is an equation that can be written in the form

$$ax + b = c$$

where a , b , and c are real numbers and $a \neq 0$ (a is not equal to 0).

Examples of linear equations in one variable: $3x - 6 = 7$, $-4m = 12$, $p = 14$

Examples of nonlinear equations in one variable: $4a^2 - 2 = 1$, $\frac{7}{q} = 3$, $|z| = 5$

Consider the linear equation $x = 5$. By inspection, we can see that the solution to this equation is 5. If we substitute 5 for x , we obtain the true statement $5 = 5$.

Now consider the equation $x - 3 = 2$. Substitute 5 for x .

$$\begin{aligned} 5 - 3 &= 2 \quad \text{True} \\ 2 &= 2 \end{aligned}$$

Equivalent equations are equations that have the same solution. $x - 3 = 2$ and $x = 5$ are equivalent equations. The idea in solving a linear equation is to get an equivalent equation with the variable by itself on one side of the equal sign and a number by itself on the other side.

The Addition Property of Equality

The same real number (or algebraic expression) may be added to both sides of an equation without changing the equation's solution. This can be expressed symbolically as:

$$\text{If } a = b, \text{ then } a + c = b + c.$$

Example: Solve and check $t - 7 = 11$.

$$\begin{aligned} t - 7 &= 11 \\ t - 7 + 7 &= 11 + 7 \\ t &= 18 \end{aligned}$$

$$\begin{aligned} \text{Check: } t &= 18 \\ t - 7 &= 11 \\ 18 - 7 &= 11 \\ 11 &= 11 \quad \checkmark \end{aligned}$$

The solution is 18.

Example: Solve and check $k + 2.3 = 5.6$.

$$k + 2.3 = 5.6$$

$$k + 2.3 - 2.3 = 5.6 - 2.3$$

$$k = 3.3$$

Check: $k = 3.3$

$$k + 2.3 = 5.6$$

$$3.3 + 2.3 = 5.6$$

$$5.6 = 5.6 \checkmark$$

The solution is 3.3.

Example: Solve and check $-\frac{2}{3} = m - \frac{1}{2}$.

$$-\frac{2}{3} = m - \frac{1}{2}$$

$$-\frac{2}{3} + \frac{1}{2} = m - \frac{1}{2} + \frac{1}{2}$$

$$-\frac{2}{3} \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{3}{3} = m$$

$$-\frac{4}{6} + \frac{3}{6} = m$$

$$-\frac{1}{6} = m$$

Check: $m = -\frac{1}{6}$

$$-\frac{2}{3} = m - \frac{1}{2}$$

$$-\frac{2}{3} \stackrel{?}{=} -\frac{1}{6} - \frac{1}{2} \cdot \frac{3}{3}$$

$$-\frac{2}{3} = -\frac{1}{6} - \frac{3}{6}$$

$$-\frac{2}{3} = -\frac{4}{6}$$

$$-\frac{2}{3} \cdot \frac{2}{2} = -\frac{4}{6}$$

$$-\frac{4}{6} = -\frac{4}{6} \checkmark$$

The solution is $-\frac{1}{6}$.

Example: Solve and check $-2r - 4 + 3r = 6$.

$$-2r - 4 + 3r = 6$$

$$r - 4 = 6$$

$$r - 4 + 4 = 6 + 4$$

$$r = 10$$

Follow the order of operations for all checks!

Check: $r = 10$

$$-2r - 4 + 3r = 6$$

$$-2(10) - 4 + 3(10) \stackrel{?}{=} 6$$

$$-20 - 4 + 30 \stackrel{?}{=} 6$$

$$-24 + 30 \stackrel{?}{=} 6$$

$$6 = 6 \checkmark$$

The solution is 10.

Example: Solve and check $20 - 7a = 26 - 8a$.

$$20 - 7a = 26 - 8a$$

$$20 - 7a + 8a = 26 - 8a + 8a$$

$$20 + a = 26$$

$$20 + a - 20 = 26 - 20$$

$$a = 6$$

Check: $a = 6$

$$20 - 7a = 26 - 8a$$

$$20 - 7(6) \stackrel{?}{=} 26 - 8(6)$$

$$20 - 42 \stackrel{?}{=} 26 - 48$$

$$-22 = -22 \checkmark$$

The solution is 6.

Section 2.2 The Multiplication Property of Equality

The Multiplication Property of Equality

The same nonzero number (or algebraic expression) may multiply both sides of an equation without changing the solution. This can be expressed symbolically as:

If $a = b$ and $c \neq 0$, then $ac = bc$.

Example: Solve and check $\frac{x}{4} = 12$.

$$\begin{aligned}\frac{x}{4} &= 12 \\ 4 \cdot \frac{x}{4} &= 12 \cdot 4 \\ \frac{4x}{4} &= 48 \\ x &= 48\end{aligned}$$

Check: If $x = 48$,
then $\frac{x}{4} = \frac{48}{4}$
 $= 12 \checkmark$

The solution is 48.

Example: Solve and check $-9v = 81$.

$$\begin{aligned}-9v &= 81 \\ \frac{-9v}{-9} &= \frac{81}{-9} \\ v &= -9\end{aligned}$$

Check: If $v = -9$,
then $-9v = -9(-9)$
 $= 81 \checkmark$

The solution is -9.

Example: Solve and check $\frac{2}{3}w = 8$.

$$\begin{aligned}\frac{2}{3}w &= 8 \\ \frac{3}{2} \cdot \frac{2}{3}w &= \frac{3}{2} \cdot 8 \\ w &= \frac{24}{2} \\ w &= 12\end{aligned}$$

Check: $w = 12$

$$\begin{aligned}\frac{2}{3}w &= 8 \\ \frac{2}{3}(12) &\stackrel{?}{=} 8 \\ \frac{24}{3} &\stackrel{?}{=} 8 \\ 8 &= 8 \checkmark\end{aligned}$$

The solution is 12.

Example: Solve and check $9 = -\frac{3}{4}k$.

$$\begin{aligned}9 &= -\frac{3}{4}k \\ -\frac{4}{3}(9) &= -\frac{4}{3}\left(-\frac{3}{4}k\right) \\ -\frac{36}{3} &= k \\ -12 &= k\end{aligned}$$

Check: If $k = -12$,
then $-\frac{3}{4}k = -\frac{3}{4}(-12)$

$$\begin{aligned}&= \frac{36}{4} \\ &= 9 \checkmark\end{aligned}$$

The solution is -12.

We could either multiply each side -1
or divide each side by -1 in these 2 problems.

Example: Solve and check $-x = 5$.

$$-x = 5$$

$$-1(-x) = -1(5)$$

$$x = -5$$

Check: $x = -5$

$$-x = 5$$

$$-(-5) = 5$$
$$5 = 5 \checkmark$$

The solution is -5 .

Example: Solve and check $-m = -8$.

$$-m = -8$$

$$\frac{-m}{-1} = \frac{-8}{-1}$$

$$m = 8$$

Check: If $m = 8$,
then $-m = -8 \checkmark$

The solution is 8 .

Example: Solve and check $2b + 1 = 9$.

$$2b + 1 = 9$$

$$2b + 1 - 1 = 9 - 1$$

$$2b = 8$$

$$\frac{2b}{2} = \frac{8}{2}$$

$$b = 4$$

Check: $b = 4$

$$2b + 1 \stackrel{?}{=} 9$$

$$2 \cdot 4 + 1 \stackrel{?}{=} 9$$

$$8 + 1 \stackrel{?}{=} 9$$

$$9 = 9 \checkmark$$

The solution is 4 .

Example: Solve and check $3r - 2 = 9$.

$$\begin{aligned}3r - 2 &= 9 \\3r - 2 + 2 &= 9 + 2 \\3r &= 11 \\\frac{3r}{3} &= \frac{11}{3} \\r &= \frac{11}{3}\end{aligned}$$

$$\begin{aligned}\text{Check: If } r &= \frac{11}{3}, \\ \text{then } 3r - 2 &= 3\left(\frac{11}{3}\right) - 2 \\ &= \frac{33}{3} - 2 \\ &= 11 - 2 \\ &= 9 \checkmark\end{aligned}$$

The solution is $\frac{11}{3}$.

Example: Solve and check $2z = -4z + 18$.

$$\begin{aligned}2z &= -4z + 18 \\2z + 4z &= -4z + 18 + 4z \\6z &= 18 \\\frac{6z}{6} &= \frac{18}{6} \\z &= 3\end{aligned}$$

$$\begin{aligned}\text{Check: } z &= 3 \\2z &= -4z + 18 \\2(3) &\stackrel{?}{=} -4(3) + 18 \\6 &\stackrel{?}{=} -12 + 18 \\6 &= 6 \checkmark\end{aligned}$$

The solution is 3.

Example: Solve and check $9y + 2 = 6y - 4$.

$$\begin{aligned}9y + 2 &= 6y - 4 \\9y + 2 - 6y &= 6y - 4 - 6y \\3y + 2 &= -4 \\3y + 2 - 2 &= -4 - 2 \\3y &= -6 \\\frac{3y}{3} &= \frac{-6}{3} \\y &= -2\end{aligned}$$

Check: $y = -2$

$$\begin{aligned}9y + 2 &= 6y - 4 \\9(-2) + 2 &\stackrel{?}{=} 6(-2) - 4 \\-18 + 2 &\stackrel{?}{=} -12 - 4 \\-16 &= -16 \checkmark\end{aligned}$$

The solution is -2 .

Example: Solve and check $-3n - 2 = -5 - 4n$.

$$\begin{aligned}-3n - 2 &= -5 - 4n \\-3n - 2 + 4n &= -5 - 4n + 4n \\n - 2 &= -5 \\n - 2 + 2 &= -5 + 2 \\n &= -3\end{aligned}$$

Check: $n = -3$

$$\begin{aligned}-3n - 2 &= -5 - 4n \\-3(-3) - 2 &\stackrel{?}{=} -5 - 4(-3) \\9 - 2 &\stackrel{?}{=} -5 + 12 \\7 &= 7 \checkmark\end{aligned}$$

The solution is -3 .