MTH 60

Section 3.6 Linear Inequalities in Two Variables

Determine whether each ordered pair is a solution of the inequality.

$3x-5y\geq -12:(2,-3),(2,8),(0,0)$

$(2,-3)$

$3x-5y\geq -12$

$3(2)-5(-3)\geq -12$

$6+15\geq -12$

$21\geq -12$

$(2,-3)$ is a solution.

$(2,8)$

$3x-5y\geq -12$

$3(2)-5(8)\geq -12$

$6-40\geq -12$

$-34\geq -12$

$(2,8)$ is not a solution.

$(0,0)$

$3x-5y\geq -12$

$3(0)-5(0)\geq -12$

$0\geq -12$

$(0,0)$ is a solution.

→ we could solve for y. $2x+y-2x > 6-2x$

$y > -2x+6$

Plot $2x + y > 6$.

↑ line not included in solution
draw a dotted line

$2x+y=6$

x	y
0	6
3	0
6	-6

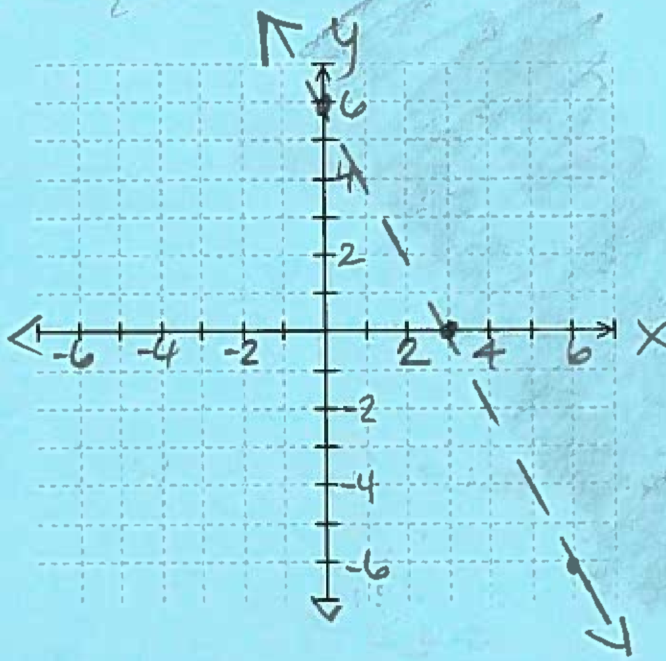
$x=6$

$2(6)+y=6$

$12+y=6$

$12+y-12=6-12$

$y=-6$



Test a point to decide which side to shade.

(0,0) $2x+y>6$

$2(0)+0>6$

$0>6$ False

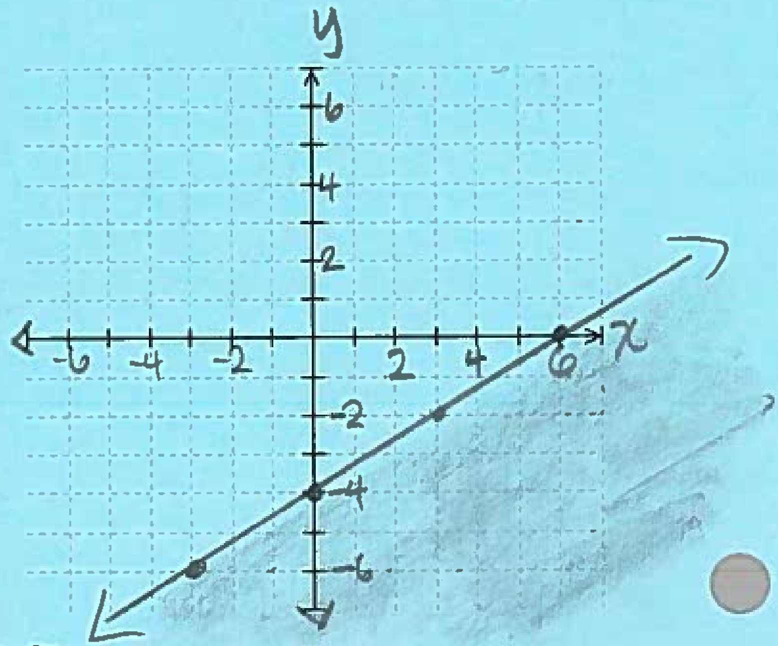
Plot $y \le \frac{2}{3}x - 4$.

$m=\frac{2}{3}$ up 2 right 3 or down 2 left 3

y-intercept: (0, -4)

line is included
draw a solid line

We want y less or equal to the line, so we shade below the line



We can always do a test point not on the line

(0,0) $y<\frac{2}{3}x-4$

$0<0-4$

$0<-4$ False

(0,0) is not in the solution set

solid line

Plot $y \le \frac{x}{20} - 5$.

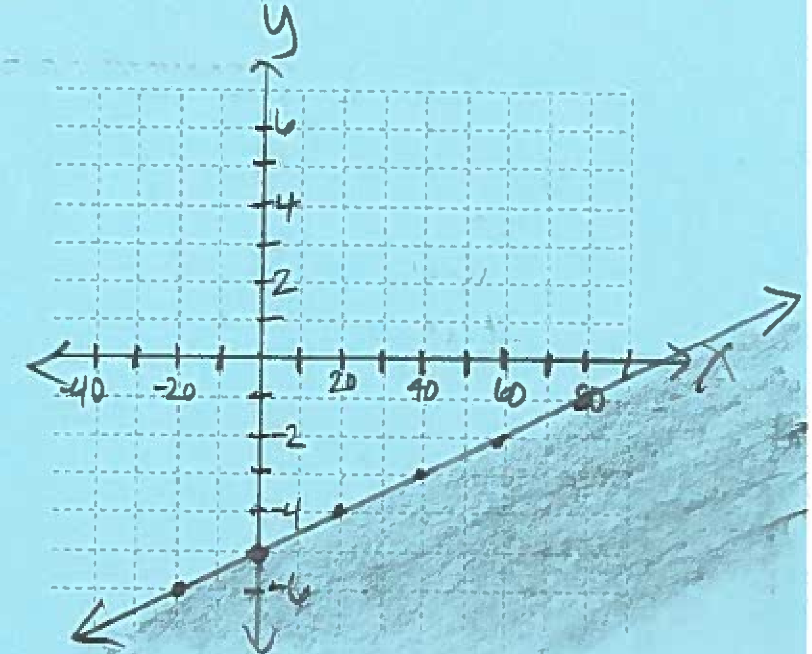
y-scale 1

x-scale 10

$m = \frac{1}{20}$ up 1 right 20

y-intercept: (0, -5)

shade below since y is less than or equal to the line.



we can always test a point off the line.

(0,0) $y \le \frac{x}{20} - 5$

$0 \le 0 - 5$

$0 \le -5$ False. (0,0) is not solution.

Write the following sentence as a linear inequality in two variables. Then graph the inequality.
The difference between the *x*-variable and the *y*-variable is at least 3.

The line is included draw a solid line

$x - y \ge 3$

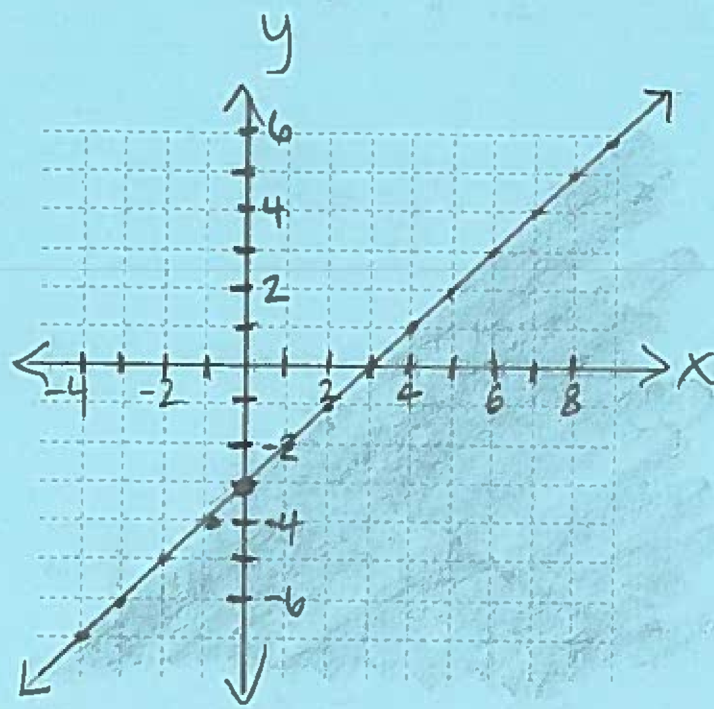
$x - y - x \ge 3 - x$

$-y \ge -x + 3$

$\frac{-y}{-1} \le \frac{-x+3}{-1}$

$y \le \frac{-x}{-1} + \frac{3}{-1}$

$y \le x - 3$



$m = 1$

$= \frac{1}{1}$ up 1 right 1

y-intercept: (0, -3)

Test (0,0) in the original inequality. $x - y \ge 3$

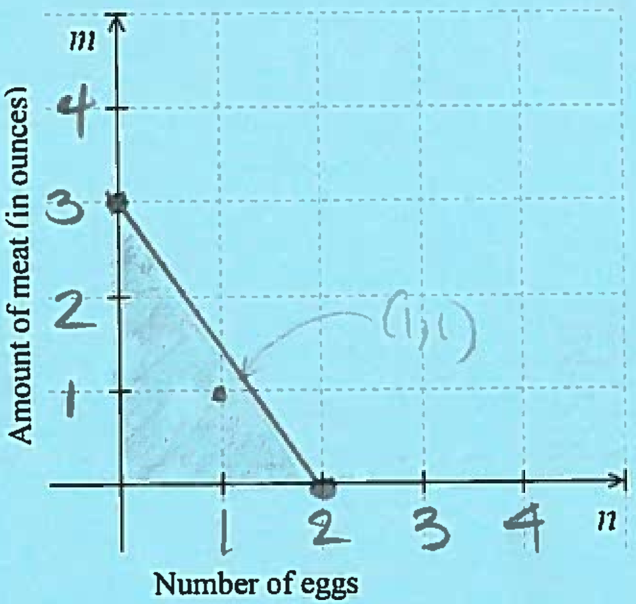
$0 - 0 \ge 3$

A patient is not allowed to have more than 330 milligrams of cholesterol per day from a diet of eggs and meat. Each egg provides 165 mg of cholesterol and each ounce of meat provides 110 mg of cholesterol.

a. Write an inequality that describes the patient's dietary restrictions where n is the number of eggs that patient eats and m represents the amount of meat (in ounces) the patient eats.

$165n + 110m \leq 330$

b. Graph the inequality. Since n and m must be nonnegative, limit the graph to quadrant I and its boundary only.



n	m
0	3
2	0

$165n + 110m - 165n \leq 330 - 165n$

$110m \leq 330 - 165n$

$\frac{110m}{110} \leq \frac{330 - 165n}{110}$

$m \leq \frac{330}{110} - \frac{165n}{110}$

$m \leq 3 - \frac{5 \cdot 33n}{5 \cdot 22}$

$m \leq 3 - \frac{33n}{22}$

$m \leq 3 - \frac{3}{2}n$

$m \leq -\frac{3}{2}n + 3$

c. Select an ordered pair satisfying the inequality. What are its coordinates and what do they represent in this situation?

(1,1)

If the patient eats 1 egg and 1 ounce of meat s/he will consume less than 330 mg. of cholesterol.

A wallet contains $5 and $10 bills. There are 15 bills in the wallet with a total value of $120. Determine the number of $5 bills and the number of $10 bills in the wallet. Define your variable, set up and solve an algebra equation, and write your answer in a sentence.

Let n represent the number of $5 bills in the wallet.

Then there are 15 − n $10 bills

$120 = 5n + 10(15-n)$

$120 = 5n + 150 - 10n$

$120 = -5n + 150$

$120 - 150 = -5n + 150 - 150$

$-30 = -5n$

$\frac{-30}{-5} = \frac{-5n}{-5}$

$6 = n$

$10 bills

$15 - n = 15 - 6$

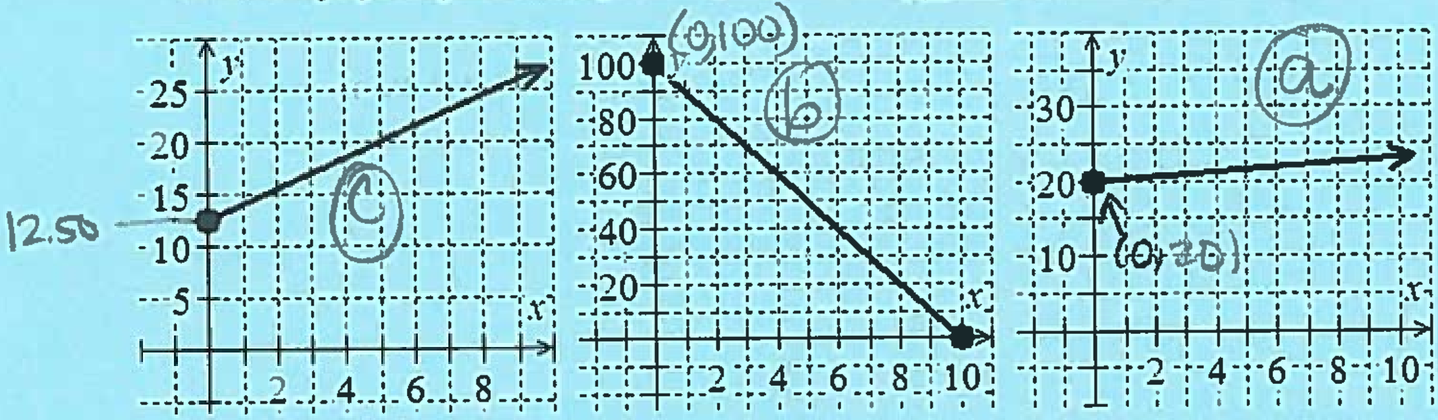
$= 9$

There are 6 $5 bills and 9 $10 bills.

Some more linear modeling problems

1. Match each description with its graph. In each case, tell what the slope represents.

   a. A sales representative receives $20 per day for food, plus $0.32 for each mile driven.
   b. A person is paying $10 per week to a friend to repay a $100 loan.
   c. An employee is paid $12.50 per hour plus $1.50 for each unit produced per hour.



a. The slope 0.32 $\frac{\$}{\text{mile}}$ which means the sales representative's reimbursement increases at a rate of 32 $\frac{\text{cents}}{\text{mile}}$.

b. The slope is -10 $\frac{\$}{\text{week}}$ which means his debt decreases at a rate of 10 $\frac{\$}{\text{week}}$.

c. The slope is 1.50 $\frac{\$}{\text{unit}}$ which means the employee's hourly wage increases at a rate of 1.50 $\frac{\$}{\text{unit}}$ produced.

$70{,}000 \frac{\$}{\text{year}}$ slope $y - y_1 = m(x - x_1)$

2. Between the years of 1990 and 2000, the annual profit for the Alpha Company increased at a rate of $70,000 per year. In 1998, the company had an annual profit of $2,000,000. Write the equation in slope-intercept form that gives the annual profit, P, for the alpha company in terms of t where t represents the time (in years) since 1990, that is, $t = 0$ corresponds to 1990. (t, P)

$(8, 2000000)$

$P - 2000000 = 70000(t - 8)$

$P - 2000000 = 70000t - 560000$

$P - 2000000 + 2000000 = 70000t - 560000 + 2000000$

$P = 70000t + 1440000$

3. A mountain climber is scaling a 400-foot cliff. The climber starts at the bottom at 10:00 am ($t = 0$) and climbs at a constant rate of 124 feet per hour.

   a. What is the slope in the linear model for this situation? Don't forget the unit.

   $124 \frac{ft}{hr}$

   b. The vertical-intercept represents the height at which the climber begins scaling the cliff. What is the vertical-intercept in the linear model?

   $(0,0)$

   c. Use the slope and vertical-intercept to write the linear model for the distance, D (in feet), that the climber climbs in terms of time t (in hours) since he started. Use slope-intercept form.

   $D = 124t$

   d. Has he reached the top at 1:00 pm? $t = 3$ $D = 124(3) = 372$ He hasn't reached the top at 1:00pm.

   e. Use your model to determine the time that the climber will reach the top of the cliff.

   $D = 400$

   $400 = 124t$

   $\frac{400}{124} = \frac{124t}{124}$

   $3.23 \approx t$

   The climber reaches the top a little after 1:00pm (at about 1:13pm).

$y - y_1 = m(x - x_1)$

4. From 1994 through 1997, the cost of owning and operating a car per mile, which includes car maintenance and repair, increased at a rate of about 2.2 cents per year. In 1995, it cost about 48.9 cents per mile to own and operate a car. Let $t = 0$ correspond to the year 1994.

   a. What is the slope of the linear equation that models this situation? Include the unit.

   $2.2 \frac{\text{cents}}{\text{year}}$

   b. Name one point on the line.

   (1, 48.9)

   c. Use the slope and the point to write a linear model for the cost, C (in dollars), of owning and operating an automobile in terms of time, t.

   (t, C)

   $C - 48.9 = 2.2(t - 1)$

   $C - 48.9 = 2.2t - 2.2$

   $C - 48.9 + 48.9 = 2.2t - 2.2 + 48.9$

   $C = 2.2t + 46.7$

   d. Use your model to predict the cost of owning and operating a car in 2003.

   $t = 9$ $C = 2.2(9) + 46.7$

   $= 66.5$

   According to the model it cost 66.5 cents per mile to own and operate a car in 2003.

5. A school club visits a science museum. Student tickets cost \$5 each. Non-student tickets cost \$7 each. The club paid \$315 for tickets. If T represents the number of student tickets and N represents the number of non-student tickets, write a linear model for this situation and then complete the table below.

   $5T + 7N = 315$

   $T = 0$ $7N = 315$

   $T = 7$ $35 + 7N = 315$

   $7N = 280$

   $T = 14$

   $5(14) + 7N = 315$

   $70 + 7N = 315$

   $7N = 245$ $N = 35$

   $\frac{7}{-5}$ $\frac{14}{-10}$

T	0	7	14	28	35	56	63
N	45	40	35	25	20	5	0

   What is the T-intercept and what does it represent?

   (T, N)

   (63, 0)

   If 63 student tickets were purchased, then 0 non-student tickets were purchased.

$y - y_1 = m(x - x_1)$

6. The average lifespan of American women has been tracked. In 1965, the average lifespan of an American woman was 74 years. In 1980, it was 77 years. A linear equation can be used to model this situation. If t represents the time (in years) since 1960 and A represents the average lifespan of an American woman (in years), write this linear equation. [Hint: You will need to write your ordered pairs, calculate the slope, write an equation in point-slope form, and then write the equation in slope-intercept form.]

$t=0$ corresponds to 1960

(t, A)

$(5, 74)$

$(20, 77)$

$$m = \frac{77-74}{20-5} = \frac{3}{15} = \frac{1}{5}$$

$m = \frac{1}{5}$ point: $(5, 74)$

$$A - 74 = \frac{1}{5}(t-5)$$

$$A - 74 = \frac{1}{5}t - 1$$

$$A - 74 + 74 = \frac{1}{5}t - 1 + 74$$

$$A = \frac{1}{5}t + 73$$

*Use the model to predict an American woman's average lifespan in 2016.

$t = 56$

$$A = \frac{1}{5}(56) + 73$$

$$= 11.2 + 73$$

$$= 84.2$$

According to the model, the average lifespan of an American woman is 84.2 years.