Section 3.4 The Slope-Intercept Form of the Equation of a Line (continued)

Plot $-2x+10y=90$. Begin by solving the equation for y.

$-2x+10y+2x=90+2x$

$10y=90+2x$

$\frac{10y}{10}=\frac{90+2x}{10}$

$y=\frac{90}{10}+\frac{2x}{10}$

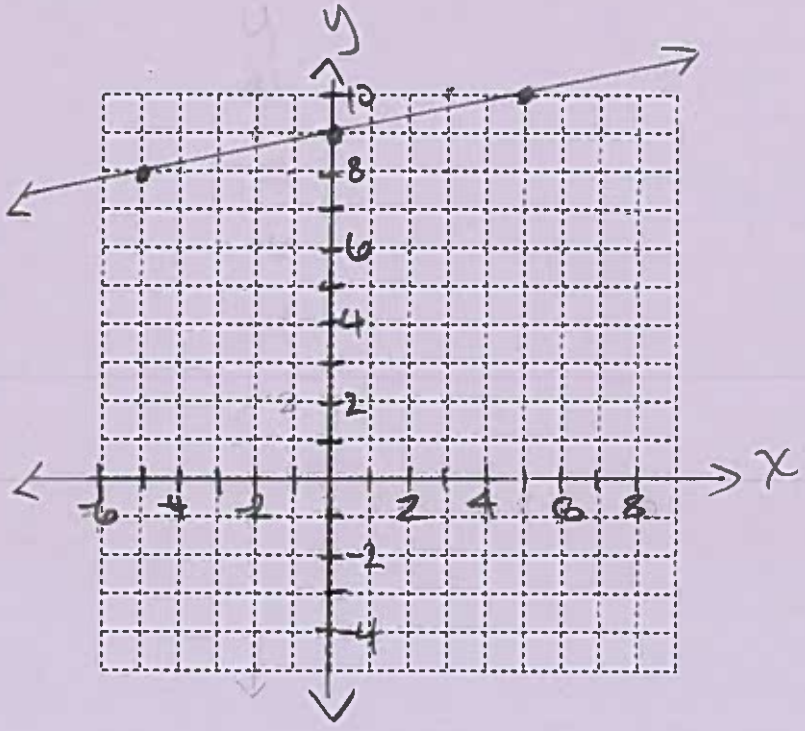
$y=9+\frac{1}{5}x$

$y=\frac{1}{5}x+9$

slope $m=\frac{1}{5}$ $\frac{\text{rise}}{\text{run}}$ up 1 right 5

y-intercept: (0, 9)

$\frac{1}{5}=\frac{-1}{-5}$ down 1 left 5



$\frac{-3}{20}$ $\frac{3}{-20}$

Plot $3t + 20N = 100$. Begin by solving the equation for N.

$3t + 20N - 3t = 100 - 3t$

$20N = 100 - 3t$

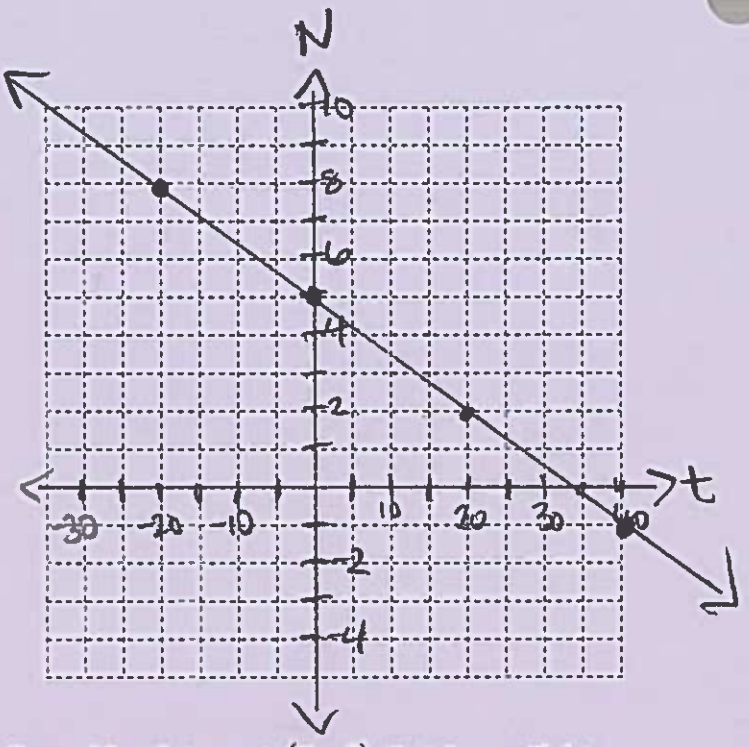
$\frac{20N}{20} = \frac{100 - 3t}{20}$

$N = \frac{100}{20} - \frac{3t}{20}$

$N = 5 - \frac{3t}{20}$

$N = -\frac{3t}{20} + 5$

slope $m = -\frac{3}{20}$ down 3 right 20 (or up 3 left 20)



N-intercept $(0, 5)$

Write an equation in the form $y = mx + b$ of the line with y-intercept $(0, -4)$ that is parallel to the line whose equation is $2x + 4y = 7$.

same slope

Solve $2x + 4y = 7$ for y.

$2x + 4y - 2x = 7 - 2x$

$4y = 7 - 2x$

$\frac{4y}{4} = \frac{7 - 2x}{4}$

$y = \frac{7}{4} - \frac{2x}{4}$ → y-intercept $(0, \frac{7}{4})$

$y = \frac{7}{4} - \frac{x}{2}$ slope: $m = -\frac{1}{2}$

The line we want has y-intercept $(0, -4)$

Our line has equation $y = -\frac{1}{2}x - 4$

perpendicular lines have negative reciprocal slopes

Write an equation in the form $y = mx + b$ of the line with y-intercept the same as the line whose equation is $3y = 2x - 15$ and is perpendicular to the line whose equation is $y = \frac{1}{3}x + 9$.

$\frac{3y}{3} = \frac{2x-15}{3}$

$y = \frac{2x}{3} - \frac{15}{3}$

$y = \frac{2x}{3} - 5$ our line has y-intercept $(0, -5)$.

Our line has slope -3

The equation of our line is $y = -3x - 5$.

negative slope

Write the equation of a decreasing line (falls from left to right) that passes through the origin and has a second point with opposite x- and y-coordinates.

y-intercept $(0,0)$

$y = mx + b$ $b = 0$

$y = mx$ m is negative

$(5, -5)$

$(-3, 3)$

$y = -1x$

The equation of our line is $y = -x$.

Section 3.5 The Point-Slope Form of the Equation of a Line

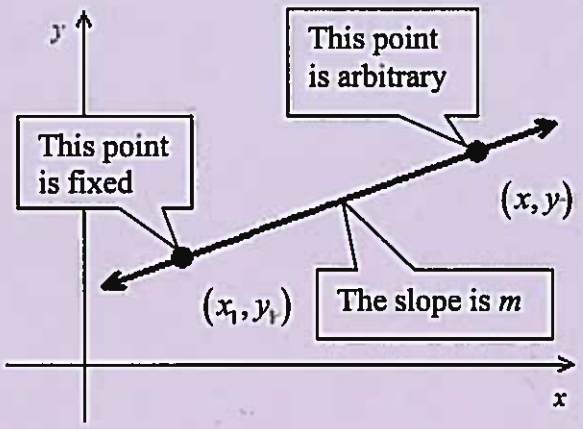
Start with the formula for slope $m=\frac{y_2-y_1}{x_2-x_1}$. Now consider a line with fixed point (x_1,y_1) and an arbitrary point (x,y). Applying the slope formula to these two points, we obtain $m=\frac{y-y_1}{x-x_1}$.

Now multiply both sides of the formula $m=\frac{y-y_1}{x-x_1}$ by $(x-x_1)$.

$m(x-x_1)=\frac{y-y_1}{x-x_1}\cdot(x-x_1)$

$m(x-x_1)=\frac{(y-y_1)(x-x_1)}{x-x_1}$

$m(x-x_1)=y-y_1$



Point-Slope Form of the Equation of a Line
The **point-slope form of the equation** of a nonvertical line with slope m that passes through the point (x_1,y_1) is $y-y_1=m(x-x_1)$.

Write the point-slope form and the slope intercept form of the equation of the line with slope 2 that passes through the point $(5,-1)$.

$m=2 \qquad (x_1,y_1)=(5,-1)$

$y-y_1=m(x-x_1)$

$y-(-1)=2(x-5)$

$y+1=2(x-5)$ point-slope form

$y+1=2x-10$

$y+1-1=2x-10-1$

$y=2x-11$ slope-intercept form

A line passes through the points $(4,-3)$ and $(-2,6)$. Find an equation of the line in point-slope form and then find the equation of the line in slope-intercept form.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{6-(-3)}{-2-4}$$

$$=\frac{6+3}{-2+(-4)}$$

$$=\frac{9}{-6}$$

$$=-\frac{3\cdot 3}{3\cdot 2}$$

$$=-\frac{3}{2}$$

Let's use $(-2,6)$

$$y-y_1=m(x-x_1)$$

$$y-6=-\frac{3}{2}(x-(-2))$$

$$y-6=-\frac{3}{2}(x+2)$$ point-slope form

$$y-6=-\frac{3}{2}x-\frac{3}{2}(2)$$

$$y-6=-\frac{3}{2}x-3$$

$$y-6+6=-\frac{3}{2}x-3+6$$

$$y=-\frac{3}{2}x+3$$ slope-intercept form

Now use the other point to write an equation of the line in point-slope form. Show that this is an equation of the same line we just found by writing it in slope-intercept form.

Let's use $(4,-3)$ now.

$$y-y_1=m(x-x_1)$$

$$y-(-3)=-\frac{3}{2}(x-4)$$

$$y+3=-\frac{3}{2}(x-4)$$ point-slope form

$$y+3=-\frac{3}{2}x-\frac{3}{2}(-4)$$

$$y+3=-\frac{3}{2}x+\frac{12}{2}$$

$$y+3=-\frac{3}{2}x+6$$

$$y+3-3=-\frac{3}{2}x+6-3$$

$$y=-\frac{3}{2}x+3$$ slope-intercept form

Table 1: Equations of Lines	
Form	What you should know
Standard Form $Ax+By=C$	Graph equations in this form using intercepts and a check point. To find the x-intercept, set $y=0$. To find the y-intercept, set $x=0$.
$y=b$	Graph equations in this form as horizontal lines with $(0,b)$ as the y-intercept.
$x=a$	Graph equations in this form as vertical lines with $(a,0)$ as the x-intercept.
Slope-Intercept Form $y=mx+b$	Graph equations in this form using the y-intercept, $(0,b)$, and the slope, m. Start with this form when writing a linear equation if you know the line's slope and y-intercept.
Point-Slope Form $y-y_1=m(x-x_1)$	Start with this form when writing a linear equation if you know the slope of the line and a point on the line other than the y-intercept or if you know two points on the line, neither of which is the y-intercept. Calculate the slope using the slope formula $m=\frac{y_2-y_2}{x_2-x_1}$. Although you begin with point-slope form, you usually solve for y and convert to slope-intercept form.

y-intercept

Find the equation of the line passing through $(0,9)$ with slope $-\frac{1}{5}$.

$y=mx+b$

$y=-\frac{1}{5}x+9$

Find the equation of the line passing through $(3,0)$ and $(3,-7)$.

$m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-7-0}{3-3}$

$=\frac{-7}{0}$

$-\frac{7}{0}$ is undefined

Vertical lines have undefined slope

The equation of our line is $X=3$.

Write an equation in slope-intercept form of the line that passes through $(5,-3)$ and is parallel to the line whose equation is $-4x+2y=7$.

parallel lines have the same slope

Solve $-4x+2y=7$ for y.

$-4x+2y+4x=7+4x$

$2y=7+4x$

$\frac{2y}{2}=\frac{7+4x}{2}$

$y=\frac{7}{2}+\frac{4x}{2}$

$y=mx+b$ $\quad y=\frac{7}{2}+2x$ $\quad m=2$

$y=2x+\frac{7}{2}$

our line has slope 2

$m=2 \quad (5,-3)$

$y-y_1=m(x-x_1)$

$y-(-3)=2(x-5)$

$y+3=2x-10$

$y+3-3=2x-10-3$

$y=2x-13$

The equation of our line is $y=2x-13$.

Write an equation in slope-intercept form of the line that passes through $(5,-9)$ and is perpendicular to the line whose equation is $x+7y=12$.

The slopes of perpendicular lines are negative reciprocals (their product is -1).

$x+7y=12$ solve for y

$x+7y-x=12-x$

$7y=12-x$

$\frac{7y}{7}=\frac{12-x}{7}$

$y=\frac{12}{7}-\frac{x}{7}$

$y=-\frac{x}{7}+\frac{12}{7}$ $\quad m=-\frac{1}{7}$

Our line has slope 7.

$m=7 \quad (5,-9)$

$y-y_1=m(x-x_1)$

$y-(-9)=7(x-5)$

$y+9=7x-35$

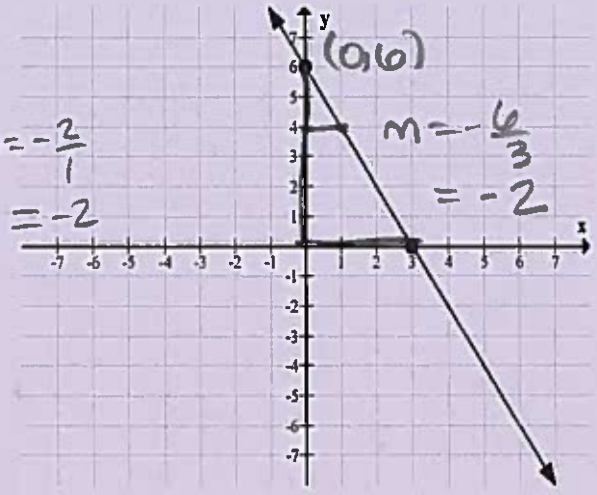
$y+9-9=7x-35-9$

$y=7x-44$

The equation of our line is $y=7x-44$.

$y = mx + b$

Write equations for the following lines.



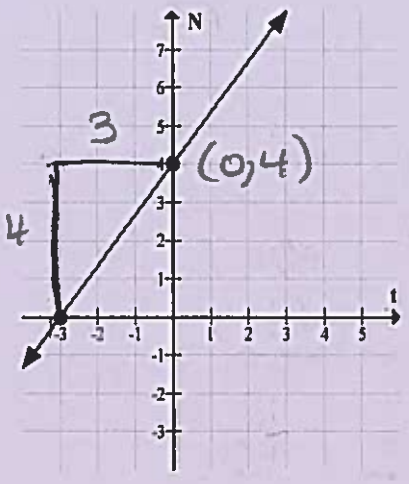
$m = -\frac{2}{1}$

$= -2$

$m = -\frac{6}{3}$

$= -2$

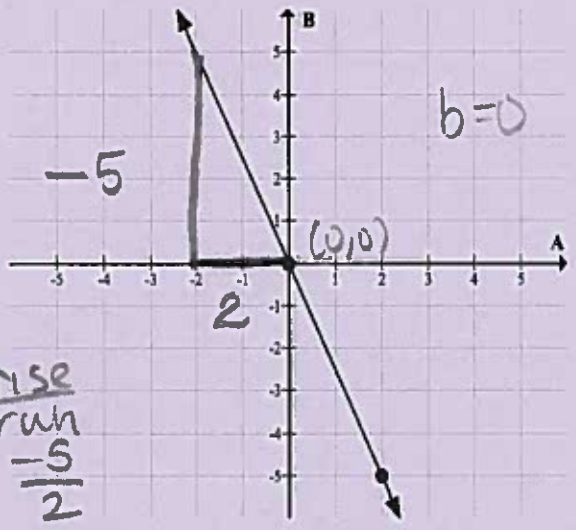
$y = -2x + 6$



$m = \frac{\text{rise}}{\text{run}}$

$= \frac{4}{3}$

$N = \frac{4}{3}t + 4$

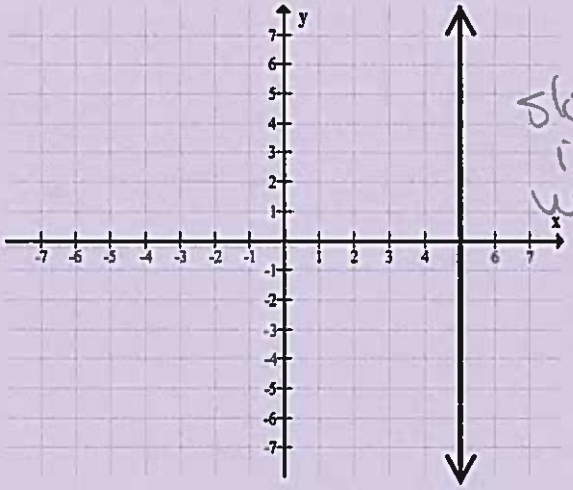


$b = 0$

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{-5}{2}$

$B = -\frac{5}{2}A$



slope is undefined

$x = 5$

MTH 60 Supplement to Section 3.3

Comparing Slope

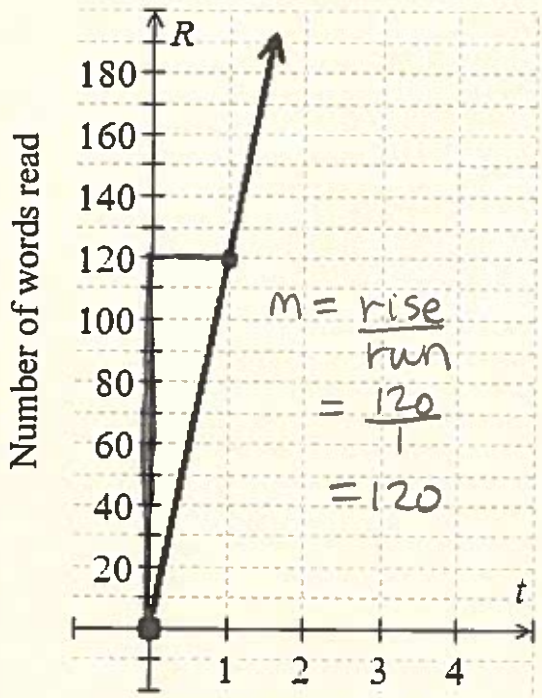


Figure 1: Ezie's reading

(t, R)

Table 1: The number of words Ashley read

t (minutes)	R (words)
0	0
3	180
6	360
9	540

3 → 180; 3 → 180; 3 → 180

Formula for the number of words Nick has read, R, at time, t, measured in minutes

$R = 100t$ $m = 100$

Determine who has the greatest reading speed and who reads at the slowest rate.

Table 1 has linear data. The slope is $\frac{180}{3} = 60$.

Ezie reads the fastest $(120 \frac{\text{words}}{\text{min}})$ and

Ashley reads the slowest $(60 \frac{\text{words}}{\text{min}})$

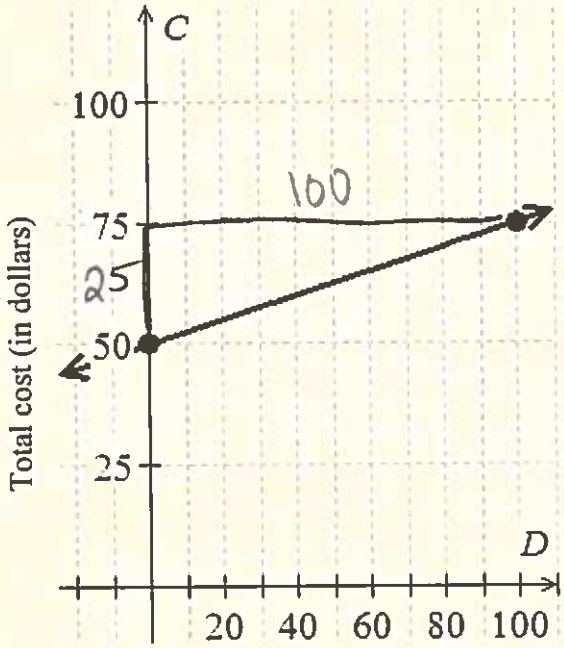


Figure 2: Car Rental

$m = \frac{\text{rise}}{\text{run}} = \frac{25}{100} = \frac{1}{4}$

Figure 2 represents the cost to rent a car. What is the vertical-intercept of the line in Figure 2?

(0, 50)

What does the vertical-intercept mean in practical terms?

The vertical-intercept tells us the car rental company has a base charge of $50 if the car travels 0 miles.

What is the slope of the line in Figure 2? Don't forget the unit. Slopes in applied problems have units.

$\frac{1}{4}\ \frac{\text{dollars}}{\text{mile}} = 0.25\ \frac{\text{dollars}}{\text{mile}}$

What does the slope mean in practical terms?

The total cost to rent the car increases at a rate of 25 $\frac{\text{cents}}{\text{mile}}$.

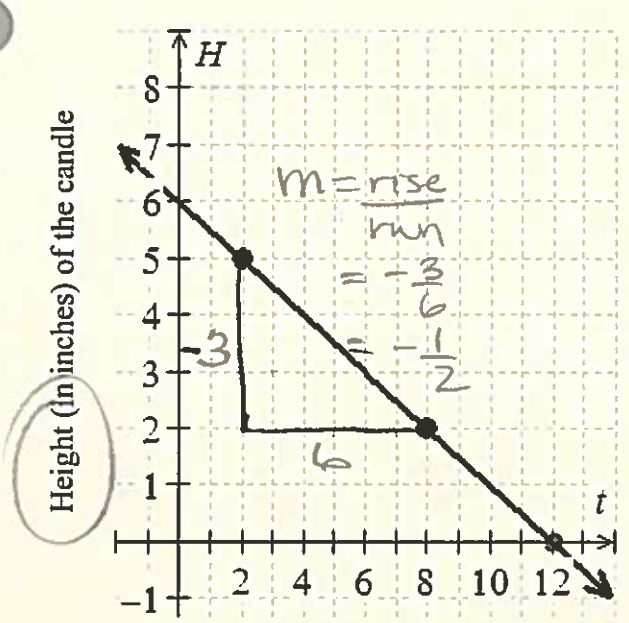


Figure 3: Height of a burning candle

Figure 3 shows the height of a candle, H, t hours after it was lit. What is the vertical-intercept of this line and what does it mean in practical terms?

The vertical intercept is (0,6) which means 0 hours since the candle was lit, the candle was 6 inches long; the candle starts out 6 inches long.

What is the horizontal intercept of this line and what does it mean in practical terms?

The horizontal intercept is (12,0) which means when the candle burns for 12 hours, it is 0 inches long; it takes 12 hours to burn out.

What is the slope of this line (include the unit) and what does it mean in practical terms?

The slope is $-\frac{1}{2}\ \frac{\text{in.}}{\text{hr}}$ which means the height of the candle decreases at a rate of $\frac{1}{2}\ \frac{\text{in.}}{\text{hr}}$.