

Section 3.3 Slope

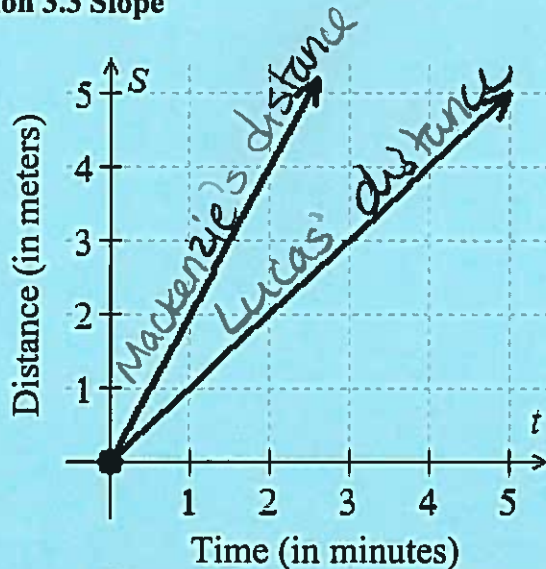


Figure 1: Swimming distances

In Figure 1, we can see that one of the lines is steeper than the other. If the steeper line represents Mackenzie's distance and the other line represents Lucas's distance, what does this tell us?

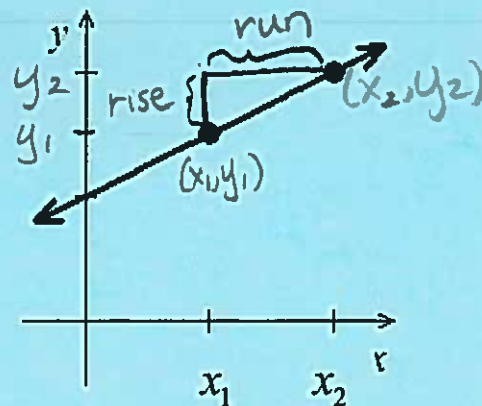
Mackenzie is a faster swimmer than Lucas.

The steepness of the line is called the **slope** of the line. We measure slope by comparing the vertical change (the **rise**) to the horizontal change (the **run**).

Definition of Slope

The **slope** of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$\begin{aligned} \frac{\text{Change in } y}{\text{Change in } x} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0 \end{aligned}$$



Example: Find the slope of the line passing through the points $(-1, -3)$ and $(2, -4)$ in two ways.

First let $(x_1, y_1) = (-1, -3)$ and $(x_2, y_2) = (2, -4)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - (-3)}{2 - (-1)} \\ &= \frac{-4 + 3}{2 + 1} \\ &= \frac{-1}{3} \end{aligned}$$

Then let $(x_1, y_1) = (2, -4)$ and $(x_2, y_2) = (-1, -3)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - (-4)}{-1 - 2} \\ &= \frac{-3 + 4}{-1 + (-2)} \\ &= \frac{1}{-3} \end{aligned}$$

$$-\frac{1}{3} = \frac{-1}{3} = \frac{1}{-3}$$

In computing slopes, it makes no difference which point you call (x_1, y_1) and which point you call (x_2, y_2) . However, you should not subtract in one order in the numerator $(y_2 - y_1)$ and then in the opposite order in the denominator $(x_1 - x_2)$.

Now, you are going to graph some lines and compare their slopes. Compute the y -values to complete the following tables and then draw each of the lines, extending them as far as possible on the provided grid. Don't forget to use a straight-edge to graph each line and draw arrows on the ends of the lines. Finally, label each line with its slope.

$y = x$	
x	y
-2	-2
-1	-1
0	0
1	1
2	2

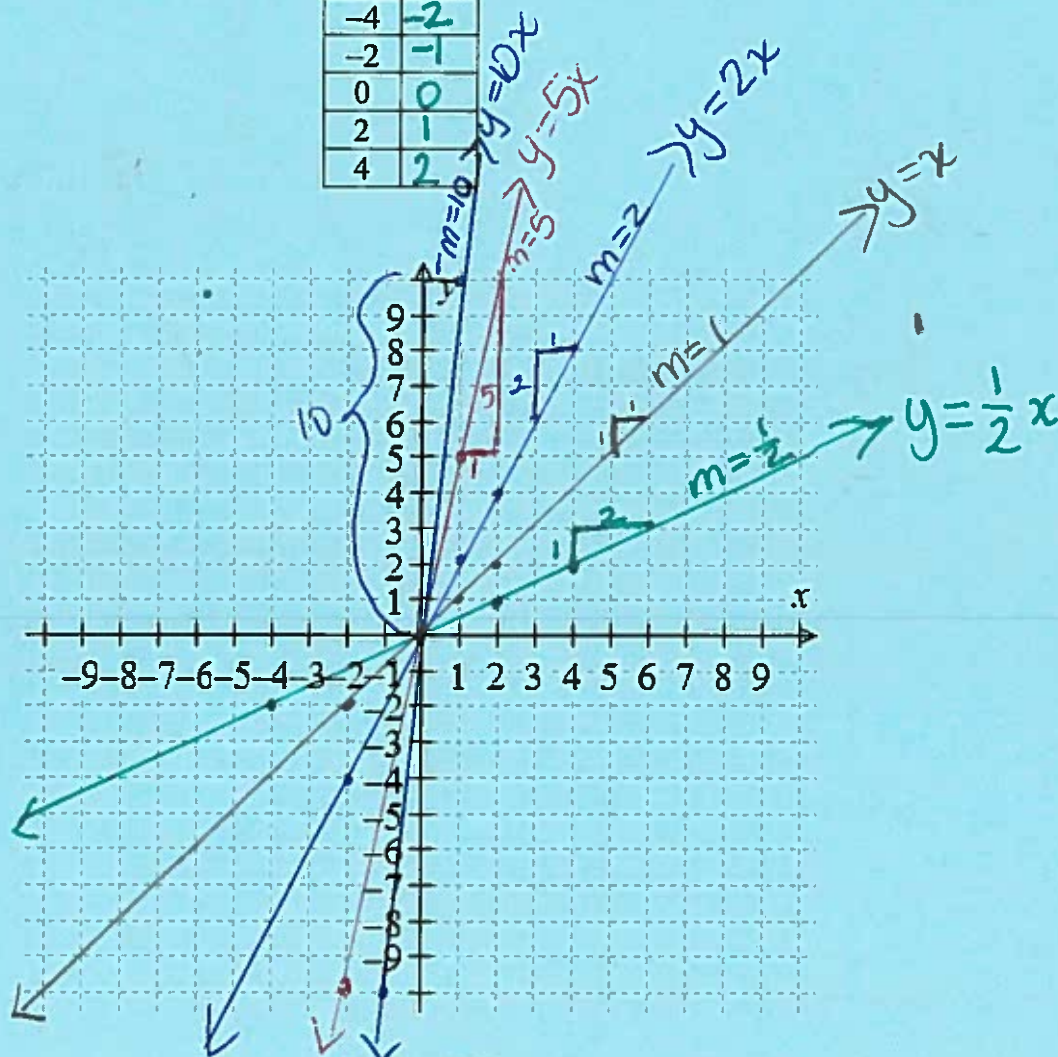
$y = 2x$	
x	y
-2	-4
-1	-2
0	0
1	2
2	4

$y = 5x$	
x	y
-2	-10
-1	-5
0	0
1	5
2	10

$y = 10x$	
x	y
-1	-10
0	0
1	10

$y = \frac{1}{2}x$	
x	y
-4	-2
-2	-1
0	0
2	1
4	2

$$m = \frac{\text{rise}}{\text{run}}$$



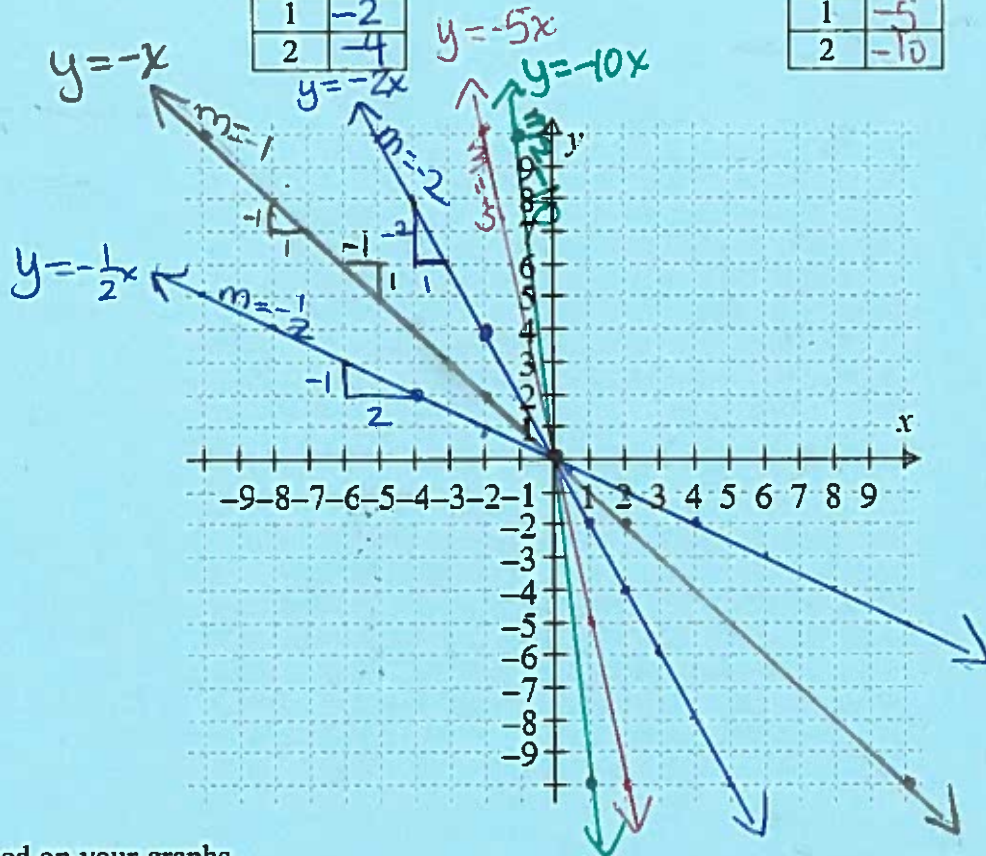
$y = -x$	
x	y
-2	2
-1	1
0	0
1	-1
2	-2

$y = -2x$	
x	y
-2	4
-1	2
0	0
1	-2
2	-4

$y = -5x$	
x	y
-2	10
-1	5
0	0
1	-5
2	-10

$y = -10x$	
x	y
-1	10
0	0
1	-10

$y = -\frac{1}{2}x$	
x	y
-4	2
-2	1
0	0
2	-1
4	-2



Fill in the blanks based on your graphs.

A positive slope means the line is going uphill (uphill or downhill).

A negative slope means the line is going downhill (uphill or downhill).

The closer the slope is to zero, the flatter (steeper or flatter) the line is.

The further away the slope is from zero, the steeper (steeper or flatter) the line is.

What is the x-intercept of each of these lines? (0,0)

What is the y-intercept of each of these lines? (0,0)

$$y = mx$$

Lines have constant slope. That is, the slope between any two points on a line is the same regardless of what two points you pick. The one exception to this rule is a vertical line which has undefined slope.

Find the slope of the line passing through the points (1,5) and (3,5).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 5}{3 - 1} \\ &= \frac{0}{2} \\ &= 0 \end{aligned}$$

horizontal lines
have zero slope

What does the graph of the line passing through the points (1,5) and (3,5) look like?

horizontal

Find the slope of the line passing through the points (2,3) and (2,4).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 3}{2 - 2} \\ &= \frac{1}{0} \end{aligned}$$

$\frac{1}{0}$ is undefined

What does the graph of the line passing through the points (2,3) and (2,4) look like?

vertical

vertical lines
have undefined
slope

Since division by zero is undefined, the slope of the vertical line in the previous example is undefined. In general, the slope of any vertical line is undefined.

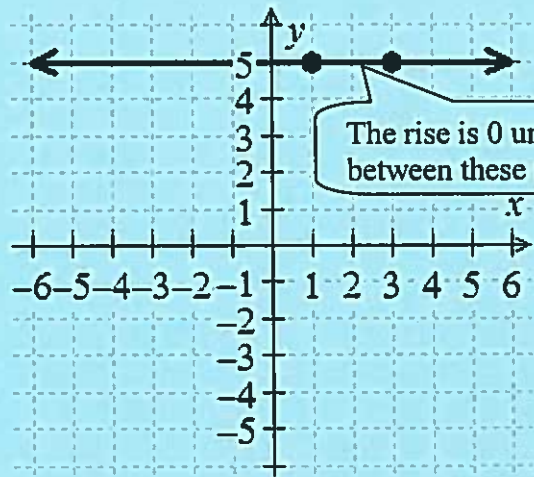


Figure 2: Horizontal lines have no vertical change

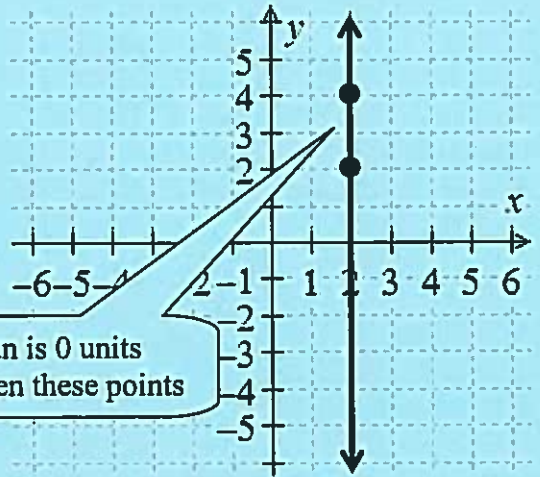


Figure 3: Vertical lines have no horizontal change

What is the slope of the line that passes through the 2 points in Figure 4?

$$m = \frac{3}{1} = 3$$

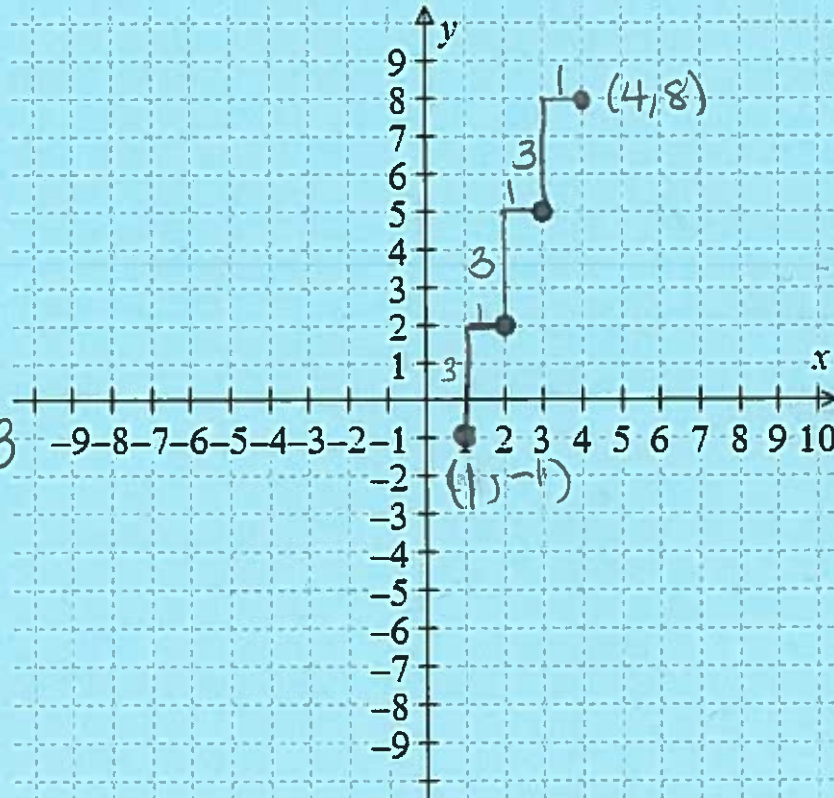


Figure 4

Find another point that creates a slope of 3 with the point (3,5).

$$(4, 8)$$

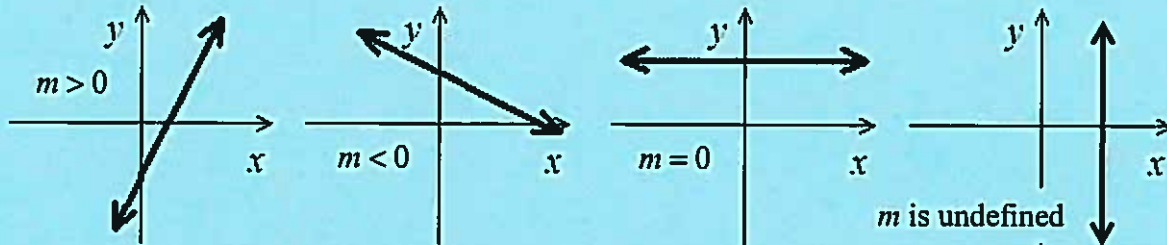
Find yet another point that creates a slope of 3 with the point (3,5).

$$(1, -1)$$

Lines with positive slope rise from left to right. We call these **increasing** lines.

Lines with negative slope fall from left to right. We call these **decreasing** lines.

Lines with zero slope are level from left to right. We call these **horizontal** lines.



Two nonintersecting lines that lie in the same plane are **parallel**.

Slope and Parallel Lines

1. If two nonvertical lines are parallel, then they have the same slope.
2. If two distinct nonvertical lines have the same slope, then they are parallel.
3. Two distinct vertical lines, each with undefined slope, are parallel.

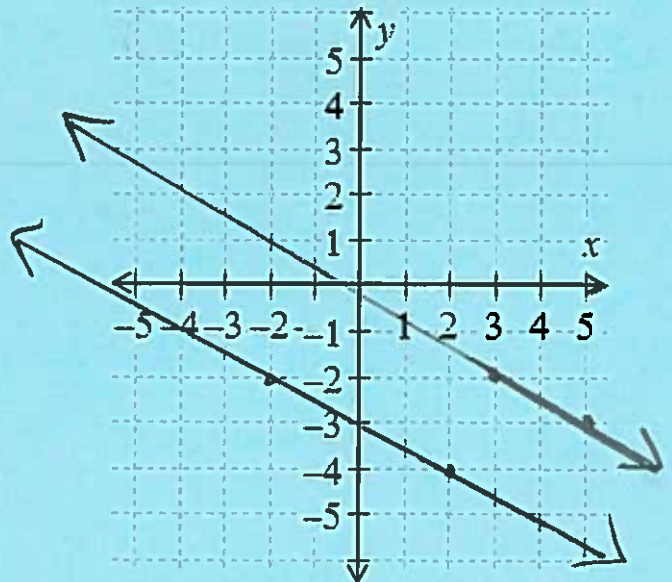
Show (using algebra) that the line passing through $(2, -4)$ and $(-2, -2)$ is parallel to the line passing through $(5, -3)$ and $(3, -2)$. Then plot the pairs of points and the lines passing through the pairs of points.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-4)}{-2 - 2} \\ &= \frac{-2 + 4}{-2 + (-2)} \\ &= \frac{2}{-4} \end{aligned}$$

$$= -\frac{1}{2}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-3)}{3 - 5} \\ &= \frac{-2 + 3}{3 + (-5)} \\ &= \frac{1}{-2} \end{aligned}$$

The slopes are the same so the lines are parallel.



Two lines that intersect at a right angle (90°) are said to be **perpendicular**.

Slope and Perpendicular Lines

1. If two nonvertical lines are perpendicular, then the product of their slopes is -1 .
2. If the product of the slopes of two lines is -1 , then the lines are perpendicular.
3. A horizontal line have zero slope is perpendicular to a vertical line having undefined slope.

Show (using algebra) that the line passing through $(0,4)$ and $(2,0)$ is perpendicular to the line passing through $(-2,0)$ and $(0,1)$. Then plot the pairs of points and the lines passing through the pairs of points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 4}{2 - 0}$$

$$= \frac{-4}{2}$$

$$= -2$$

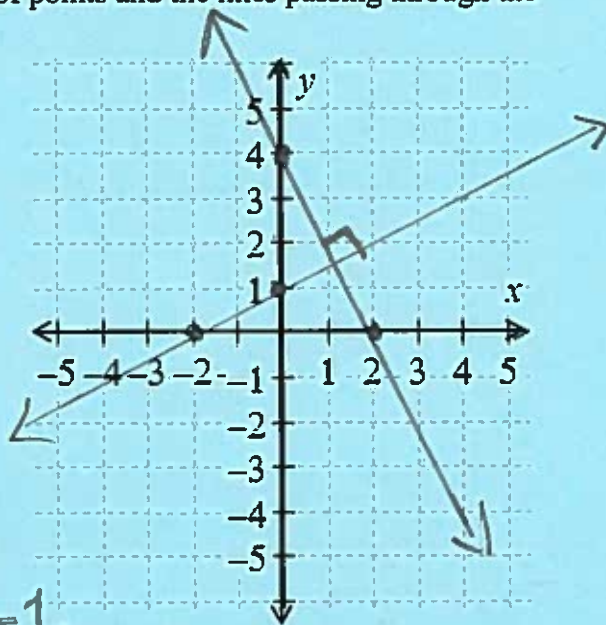
$$m = \frac{1 - 0}{0 - (-2)}$$

$$= \frac{1}{2}$$

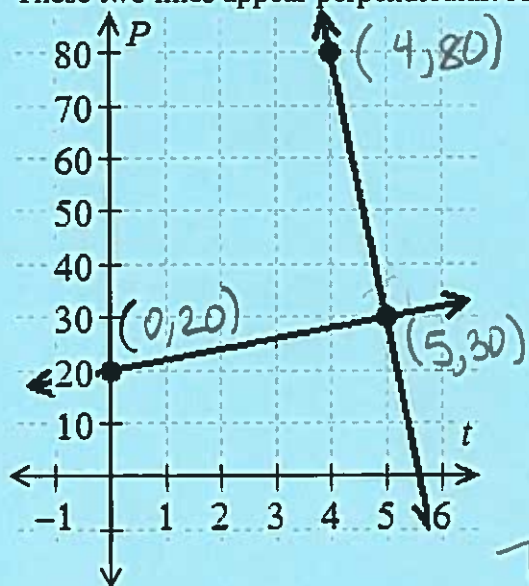
$$-2\left(\frac{1}{2}\right) = -1$$

The lines are perpendicular since the product of their slopes is -1 .

The slopes of perpendicular lines are reciprocals with opposite signs, called **negative reciprocals**. Two nonvertical lines are perpendicular if the slope of one is the negative reciprocal of the slope of the other.



These two lines appear perpendicular. Are they really perpendicular?



$$m = \frac{80 - 30}{4 - 5}$$

$$= \frac{50}{-1}$$

$$= -50$$

$$m = \frac{30 - 20}{5 - 0}$$

$$= \frac{10}{5}$$

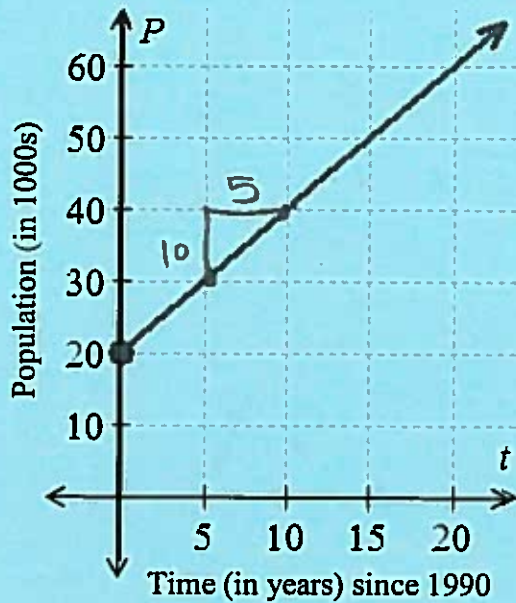
$$= 2$$

The lines are not perpendicular since the product of their slopes is not -1 .

The differing scales on the t -axis and P -axis made the lines appear perpendicular.

Slope as a Rate of Change

What is the slope of the line graphed? Don't forget the unit! Slopes in applied problems have units.

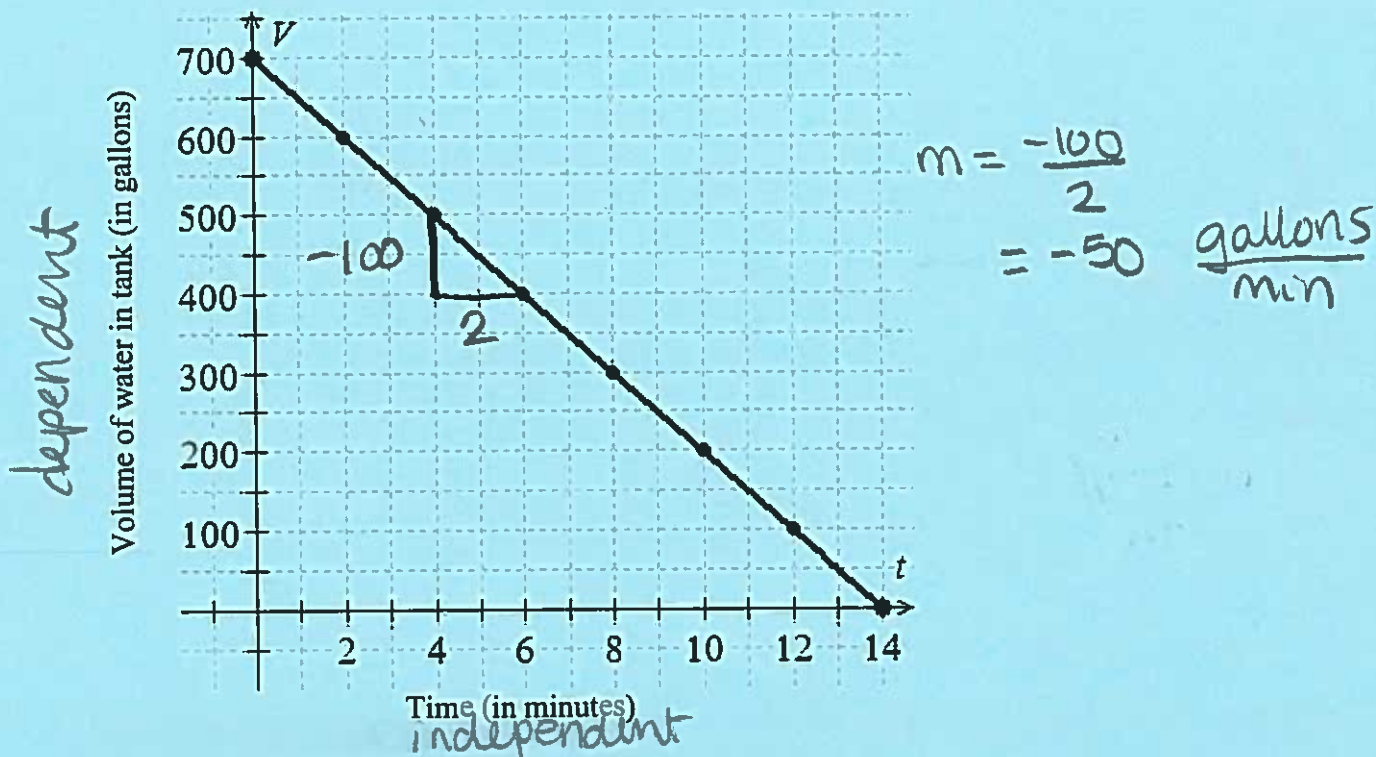


$$m = \frac{10}{5} \frac{\text{thousand people}}{\text{year}}$$
$$= 2 \frac{\text{thousand people}}{\text{year}}$$

What is the practical meaning of the slope?

The population is increasing
at a rate of $2000 \frac{\text{people}}{\text{year}}$.

What is the slope of the line graphed? Don't forget the unit! Slopes in applied problems have units.



What is the practical meaning of the slope?

The volume of water is decreasing at a rate of $50 \frac{\text{gallons}}{\text{min}}$.

The volume of water depends on the time.

The population depends on time in the previous example.

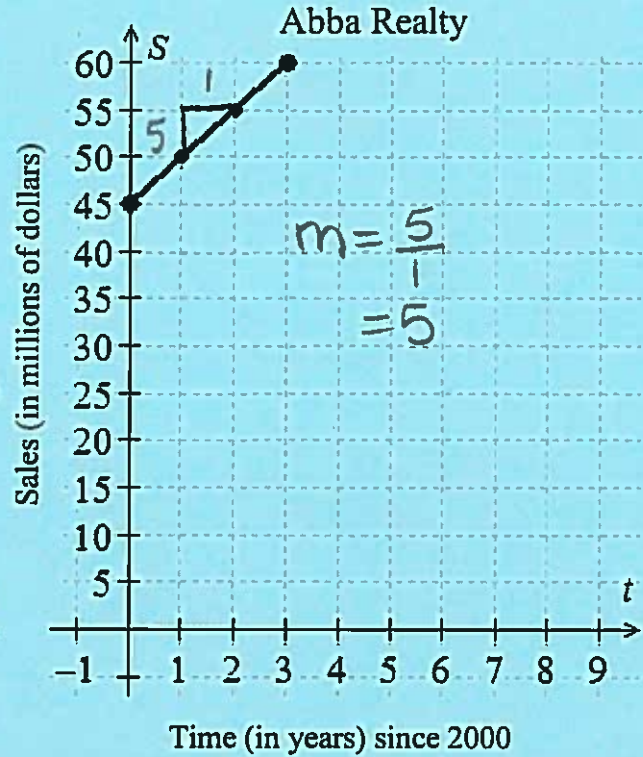
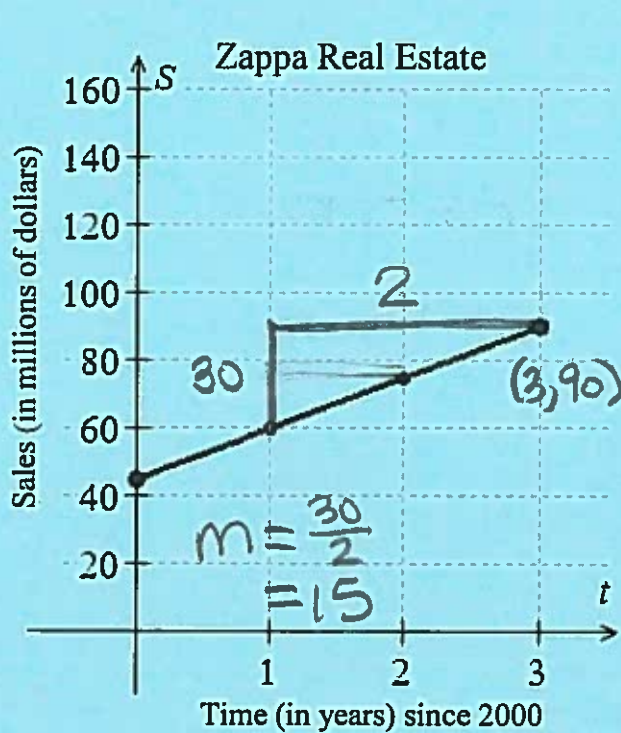
Slope can always be thought of as a rate of change. We can always interpret slope in the same manner by using the following sentences and filling in the blanks.

The _____ increases at a rate of _____.
dependent quantity slope

The _____ decreases at a rate of _____.
dependent quantity |slope|

Be careful to use the absolute value of the slope when your slope is negative as you have already said "decreases" at a rate of, which takes care of the negative.
↑ absolute value

Which company's sales are increasing at a faster rate?



Zappa real estate's sales are increasing at a faster rate since the slope of the line for Zappa Real Estate's sales is greater than the slope of the line for Abba Realty's sales.

When data is given in a horizontal table, the first row generally represents the independent variable.

Table 1: $V = 60 - 15t$

t	0	1	2	3	4
V	60	45	30	15	0

$(0, 60)$
 $(1, 45)$

Based on the data in Table 1, what is the slope of the equation $V = 60 - 15t$?

$$m = \frac{45 - 60}{1 - 0} = \frac{-15}{1} = -15$$

$$\frac{\text{rise}}{\text{run}} \quad \frac{\text{mph}}{\text{sec}}$$

Table 1: $V = 60 - 15t$

time, t , (in seconds)	0	1	2	3	4
speed, V , (in mph)	60	45	30	15	0

$$m = -15 \frac{\text{mph}}{\text{sec}}$$

Given that V represents the speed of a car t seconds after the brakes are applied, what is the practical significance of the slope?

The speed is decreasing at a rate of $15 \frac{\text{mph}}{\text{sec}}$.

Do the data in Table 2 represent a linear relationship?

Table 2

t	0	2	4	6	8	10	12
N	3	6	9	11	15	17	21

① Plot the points to see if they all lie on the same line. or

② Find the slope, making sure we have a constant slope.

The slope is not constant so the data in Table 2 is not linear.

Do the data in Table 3 represent a linear relationship?

Table 3

n	0	2	6	7	10	15	20
R	7	6	4	3.5	2	-0.5	-3

$$m = \frac{\text{rise}}{\text{run}}$$

$$-\frac{1}{2} \quad -\frac{2}{4} = -\frac{1}{2} \quad \frac{-0.5}{1} = -\frac{1}{2} \quad \frac{-1.5}{3} = -\frac{1}{2} \quad \frac{-2.5}{5} = -\frac{1}{2}$$

The slope is constant so the data Table 3 is linear.

Section 3.4 The Slope-Intercept Form of the Equation of a Line

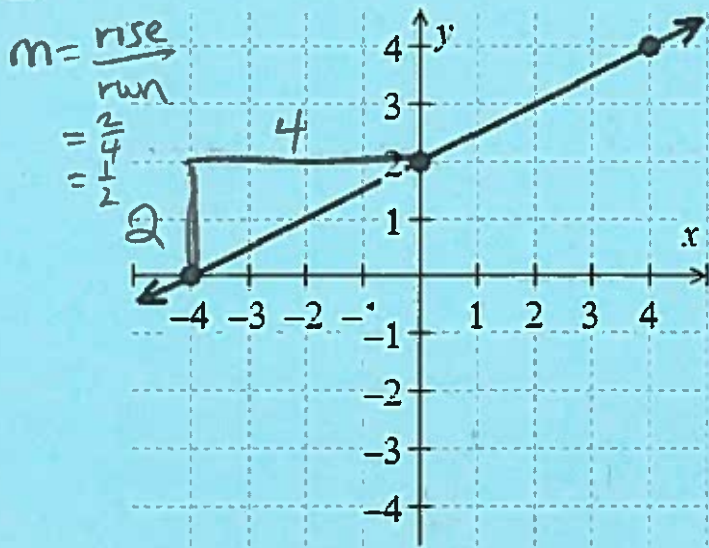


Figure 5: $y = \frac{1}{2}x + 2$

What is the slope and y-intercept of the line in Figure 1?

The slope is $\frac{1}{2}$.

The y-intercept is $(0, 2)$.

Slope-Intercept Form of the Equation of a Line

The slope-intercept form of the equation of a nonvertical line with slope m and y-intercept $(0, b)$ is $y = mx + b$.

$$m = \frac{\text{rise}}{\text{run}}$$

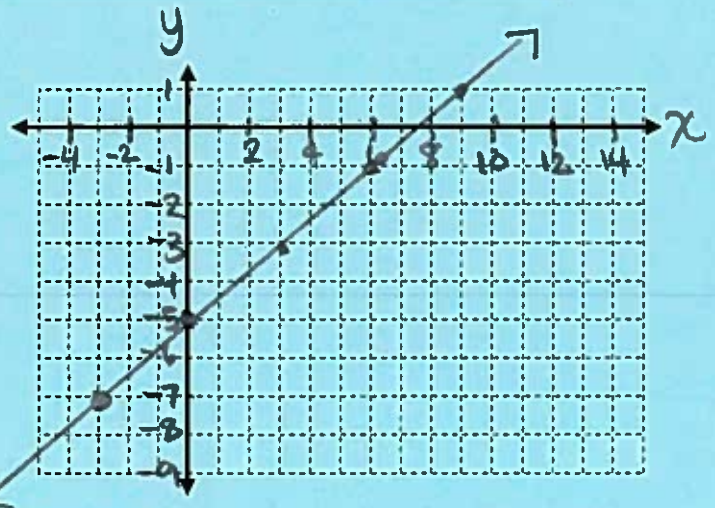
Plot $y = \frac{2}{3}x - 5$

$$m = \frac{2}{3}$$

rise up 2
run right 3

$(0, -5)$

$\frac{2}{3} = \frac{-2}{-3}$
down 2
left 3



How can you use the slope to find a point in the fourth quadrant?

third

Use the fact that $\frac{2}{3} = \frac{-2}{-3}$ and go down 2 and left 3 from the y-intercept.

Plot $3x + y = 2$. Begin by solving the equation for y .

$$3x + y - 3x = 2 - 3x$$

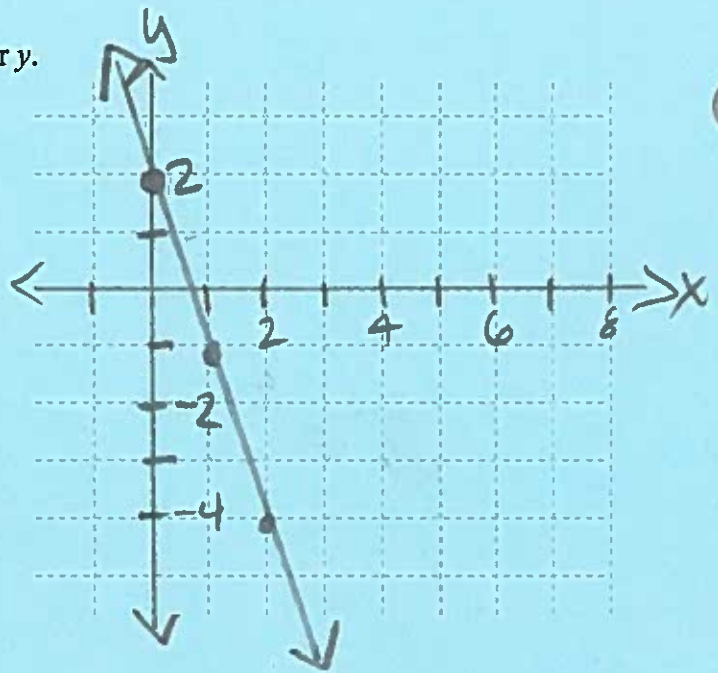
$$y = 2 - 3x$$

$$y = -3x + 2$$

$$m = -3$$

$$= -\frac{3}{1} \quad \frac{\text{down } 3}{\text{right } 1} \text{ or } \frac{\text{up } 3}{\text{left } 1}$$

$$(0, 2)$$



Plot $N = 25t - 50$

(t, N)

$$m = 25$$

$$= \frac{25}{1} \quad \frac{\text{up } 25}{\text{right } 1}$$

$$(0, -50)$$

