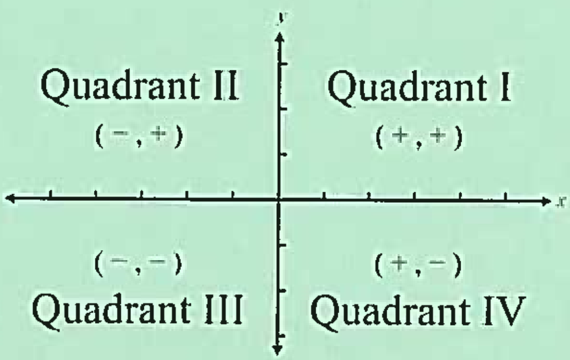
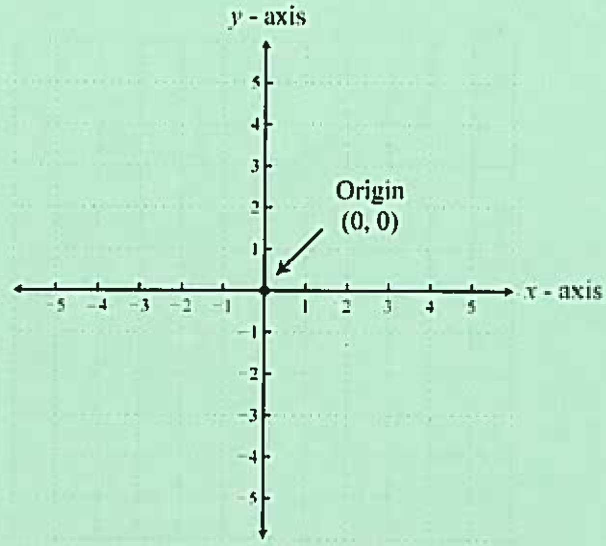
MTH 60

Section 3.1 Graphing Linear Equations in Two Variables

The rectangular coordinate system or the Cartesian coordinate system (in honor of René Descartes)





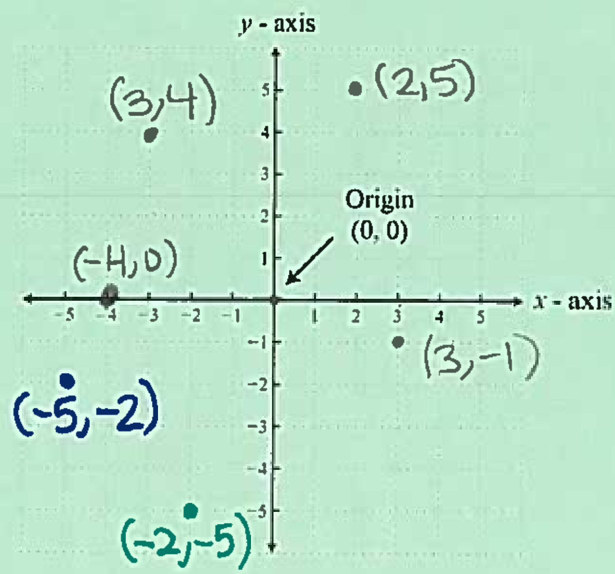
Each point in the rectangular coordinate system corresponds to an **ordered pair**. The points are called "ordered" pairs because order matters. The first number in the ordered pair corresponds to the horizontal axis and the second point in the ordered pair corresponds to the vertical axis. The origin is the point $(0,0)$, and is shown above. Plot the points $(2,5), (3,-1), (-4,0)$, and $(-3,4)$. Which quadrant does each point lie in?

$(2,5)$ lies in Quadrant I

$(3,-1)$ lies in Quadrant IV

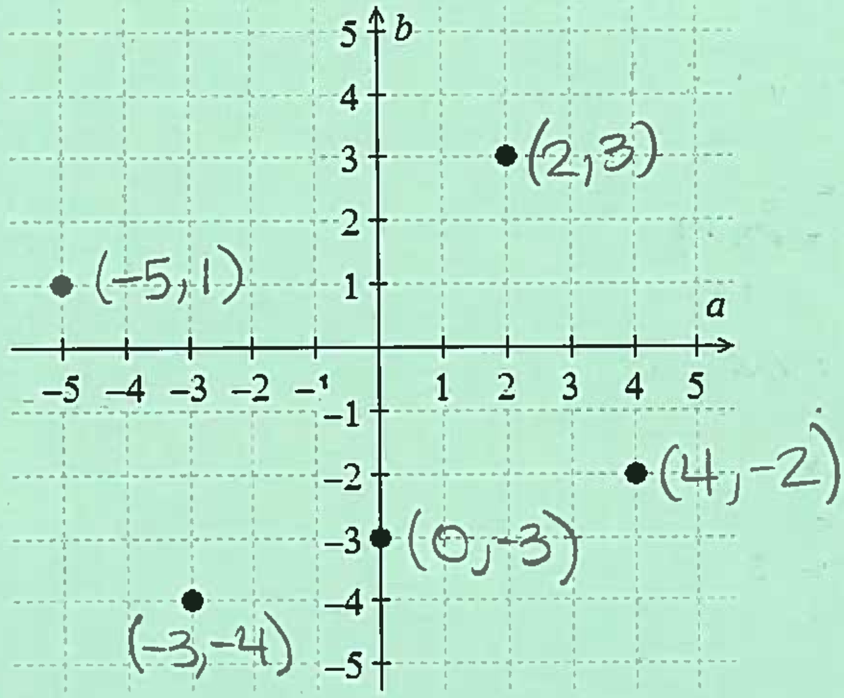
$(-4,0)$ lies on the x-axis

$(-3,4)$ lies in Quadrant II



Show that order makes a difference by plotting $(-2,-5)$ and $(-5,-2.)$

The labels on the axes can be different variables. That is why I refer to the axes as the horizontal and vertical axes as opposed to the x-axis and the y-axis. Find the coordinates of the following points. Note that with this labeling, ordered pairs have the form (a,b).



Determine whether the ordered pairs $(3,-2)$ and $(2,3)$ are solutions of $2x-3y=12$.

(x,y)

$(3,-2)$

$2x-3y=12$

$2(3)-3(-2) \stackrel{?}{=} 12$

$6+6=12$

$12=12$ ✓

$(2,3)$

$2x-3y=12$

$2(2)-3(3) \stackrel{?}{=} 12$

$4-9 \stackrel{?}{=} 12$

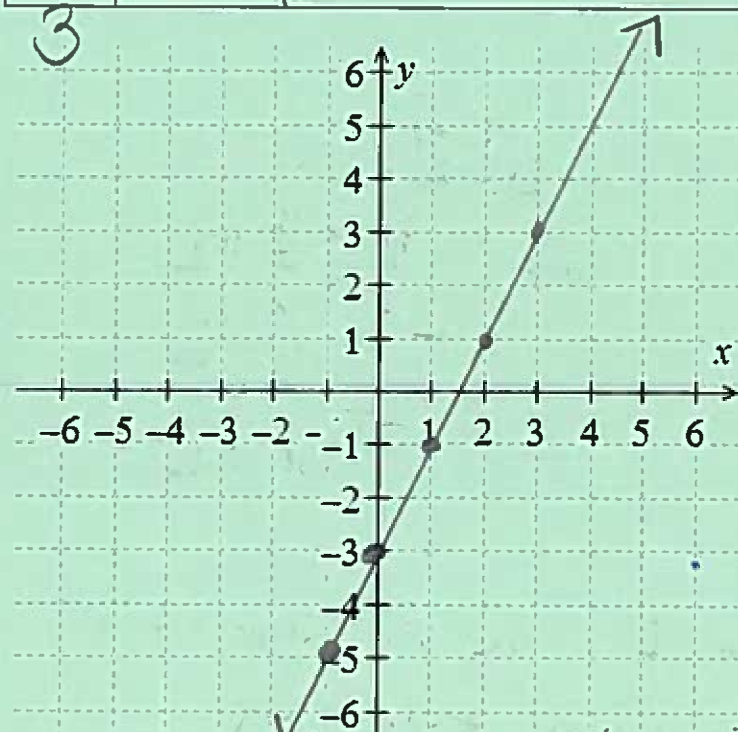
$-5 \neq 12$

$(3,-2)$ is a solution to $2x-3y=12$, but $(2,3)$ is not a solution.

Find five solutions of $y = 2x - 3$. Then plot these points.

x	$y = 2x - 3$	(x, y)
-2	$y = 2(-2) - 3$ $= -4 - 3$ $= -7$	$(-2, -7)$
-1	$y = 2(-1) - 3$ $= -2 - 3$ $= -5$	$(-1, -5)$
0	$y = 2(0) - 3$ $= 0 - 3$ $= -3$	$(0, -3)$
1	$y = 2(1) - 3$ $= 2 - 3$ $= -1$	$(1, -1)$
2	$y = 2(2) - 3$ $= 4 - 3$ $= 1$	$(2, 1)$

3 $(3, 3)$

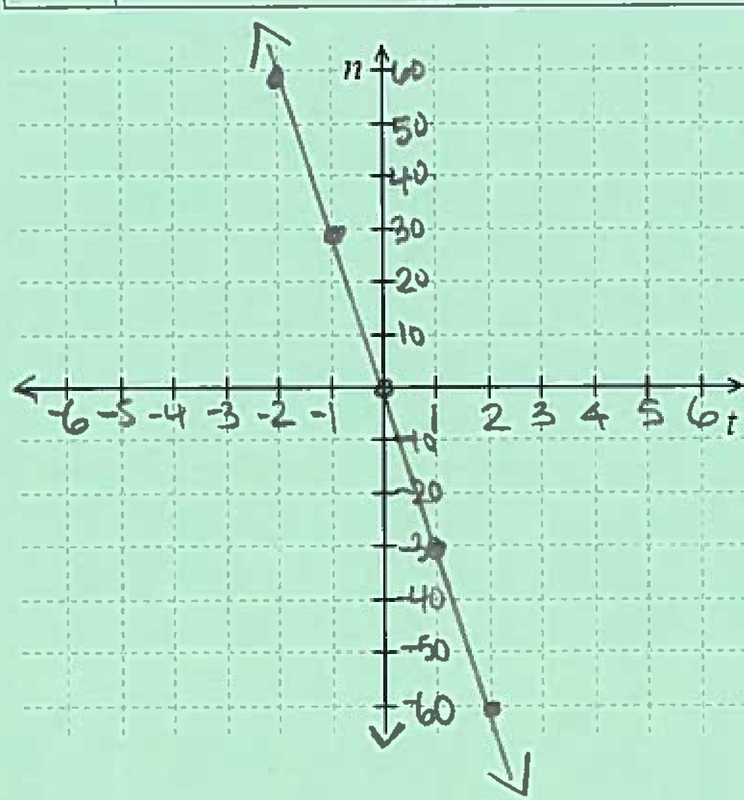


$y = 2(3) - 3$
$= 6 - 3$
$= 3$

Connect the points. What do you notice? The points form a line.

Graph the equation $n = -30t$.

t	$n = -30t$	(t, n)
-2	$n = -30(-2)$ $= 60$	$(-2, 60)$
-1	$n = -30(-1)$ $= 30$	$(-1, 30)$
0	$n = -30(0)$ $= 0$	$(0, 0)$
1	$n = -30(1)$ $= -30$	$(1, -30)$
2	$n = -30(2)$ $= -60$	$(2, -60)$

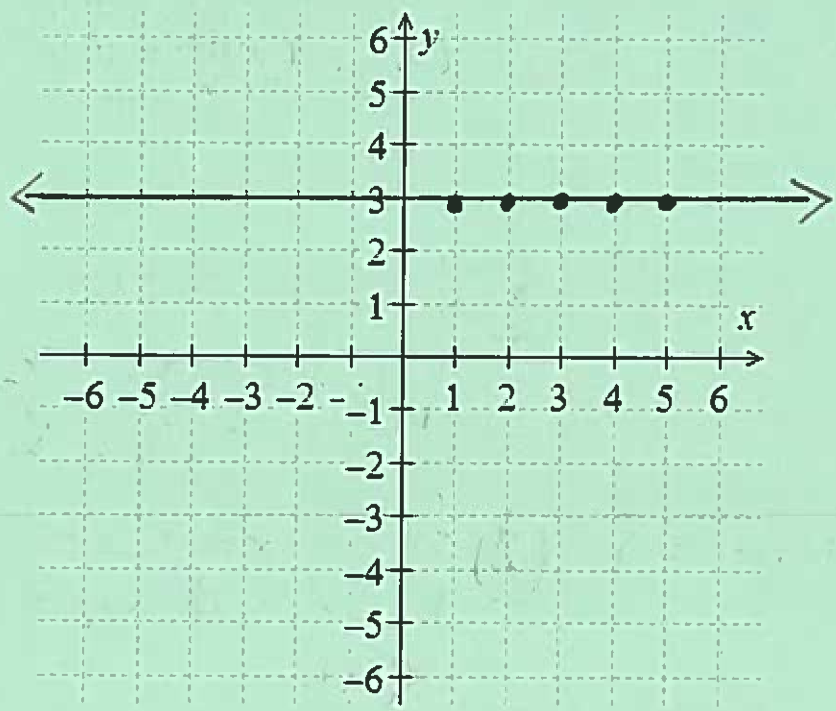


1. The scales on the axes does not have to by 1.
2. Each axis can have a different scale.
3. Choose scales so we can be accurate (no estimating).

Equations like $y = 2x - 3$ and $n = -30t$ are called **linear equations in two variables** because the graph of each equation is a line. Any equation that can be written in the form $y = mx + b$, where m and b are constants, is a linear equation in two variables. Later, we will study how the numbers m and b affect the equation's graph.

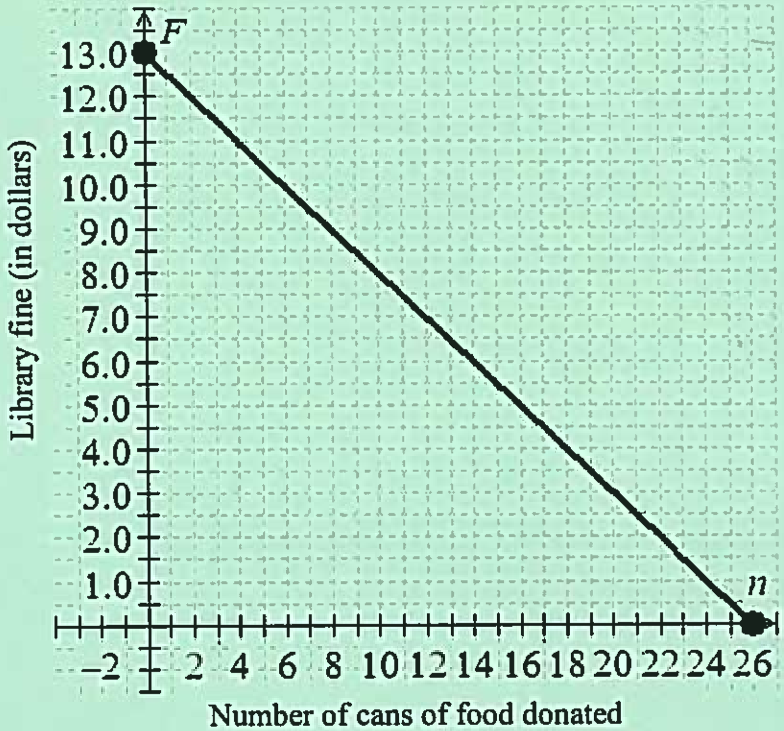
Graph the equation $y = 3$.

x	$y = 3$	(x, y)
1	3	(1, 3)
2	3	(2, 3)
3	3	(3, 3)
4	3	(4, 3)
5	3	(5, 3)



Section 3.2 Graphing Linear Equations Using Intercepts

If Brandon owed $13.00 in library fines and his local library is running a February Food for Fines event that forgives $0.50 in fines for each can of food donated, we can model this with the formula $F = -0.50n + 13$ where F represents the amount of Brandon's fine (in dollars) and n represents the number of cans of food he donates. The graph of this linear model is shown below.



(independent, dependent)
variable variable

(n, F)

Library fines depend on the number of cans of food donated.

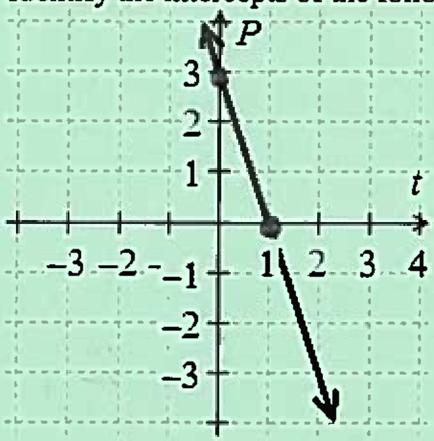
The points show where the line intersects the horizontal-axis (n-axis) and the vertical-axis (F-axis). These important points are called **intercepts**. What are these intercepts and what do they mean in practical terms?

The horizontal-intercept (n-intercept) is (26,0). This means if Brandon donates 26 cans of food his library fine is reduced to $0.

The vertical-intercept (F-intercept) is (0,13). This means that if Brandon donates 0 cans of food, his library fine is $13.

The formula $F = -0.50n + 13$ can be rewritten as $0.50n + F = 13$ if we add $0.50n$ to both sides of the equation. This equation, $0.50n + F = 13$, is in **standard form**. It's in the form $Ax + By = C$ where A, B, and C are constants and A and B are not both zero. A is called the **coefficient** of x and B is called the **coefficient** of y. Any equation that can be written in this form graphs to a line. It is useful to use intercepts to graph lines given in standard form.

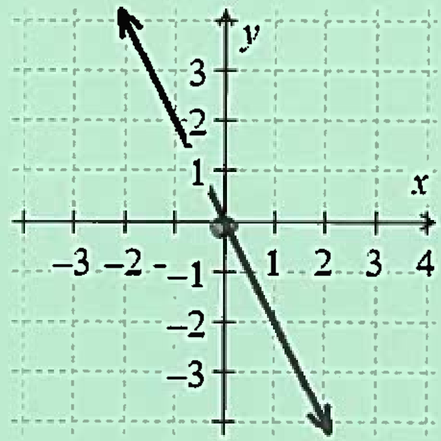
Identify the intercepts of the following graphs.



(t, P)

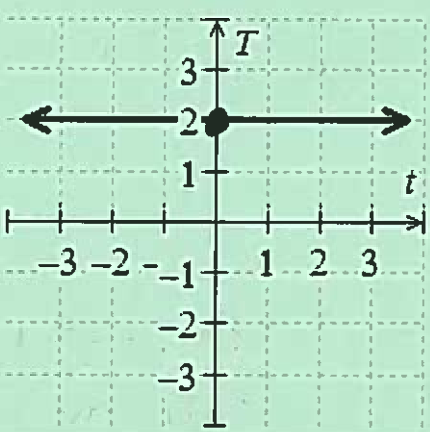
(1,0)

(0,3)



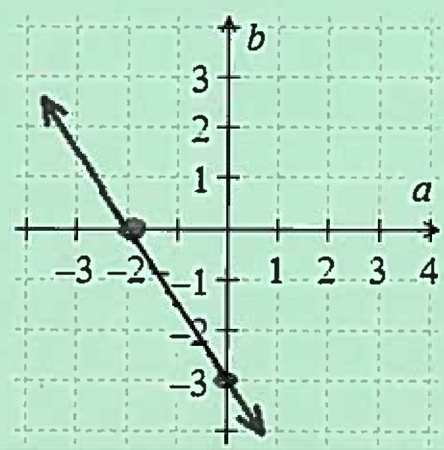
(x,y)

(0,0)



(t, T)

(0,2)



(a, b)

(−2,0)

(0,−3)

Using Intercepts to Graph $Ax + By = C$

1. Find the x-intercept. Let $y = 0$ and solve for x.
2. Find the y-intercept. Let $x = 0$ and solve for y.
3. Find a checkpoint, a third ordered-pair solution.
4. Graph the equation by drawing a line through the three points. Don't forget to use a straight-edge to draw the line and put arrows on both ends of the line to indicate the line goes on forever. Also, remember to label and scale your axes. The horizontal and vertical axes do not need to have the same scale.

numbers

variables.

Use intercepts to graph $3x + 4y = 12$.

x-intercept

$y=0$

$3x+4(0)=12$

$3x=12$

$\frac{3x}{3}=\frac{12}{3}$

$x=4$

$(4,0)$

y-intercept

$x=0$

$3(0)+4y=12$

$4y=12$

$\frac{4y}{4}=\frac{12}{4}$

$y=3$

$(0,3)$

Check point

Try $x=3$

$3(3)+4y=12$

$9+4y=12$

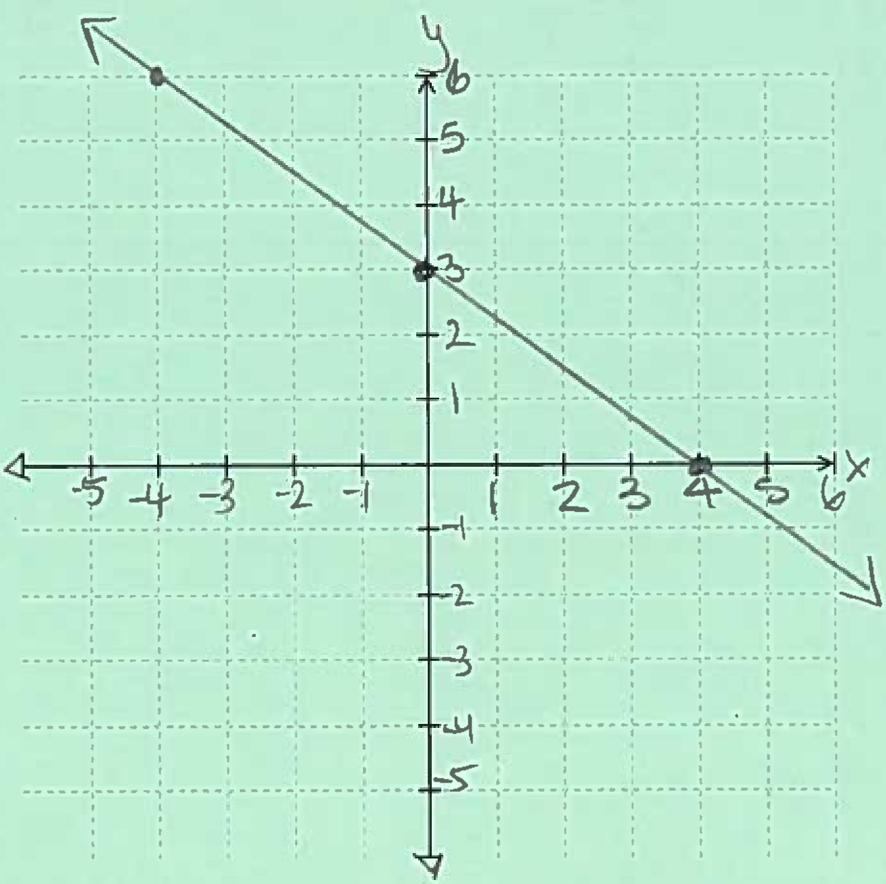
$9+4y-9=12-9$

$4y=3$

$\frac{4y}{4}=\frac{3}{4}$

$y=\frac{3}{4}$

$(3,\frac{3}{4})$



Try $x=-4$

$3x+4y=12$

$3(-4)+4y=12$

$-12+4y=12$

$-12+4y+12=12+12$

$4y=24$

$\frac{4y}{4}=\frac{24}{4}$

$y=6$

$(-4,6)$

Use intercepts to graph $20x - 3y = 60$.

x-intercept

$y = 0$

$20x - 3(0) = 60$

$20x = 60$

$\frac{20x}{20} = \frac{60}{20}$

$x = 3$

$(3, 0)$

y-intercept

$x = 0$

$20(0) - 3y = 60$

$-3y = 60$

$\frac{-3y}{-3} = \frac{60}{-3}$

$y = -20$

$(0, -20)$

check point

$x = -3$

$20(-3) - 3y = 60$

$-60 - 3y = 60$

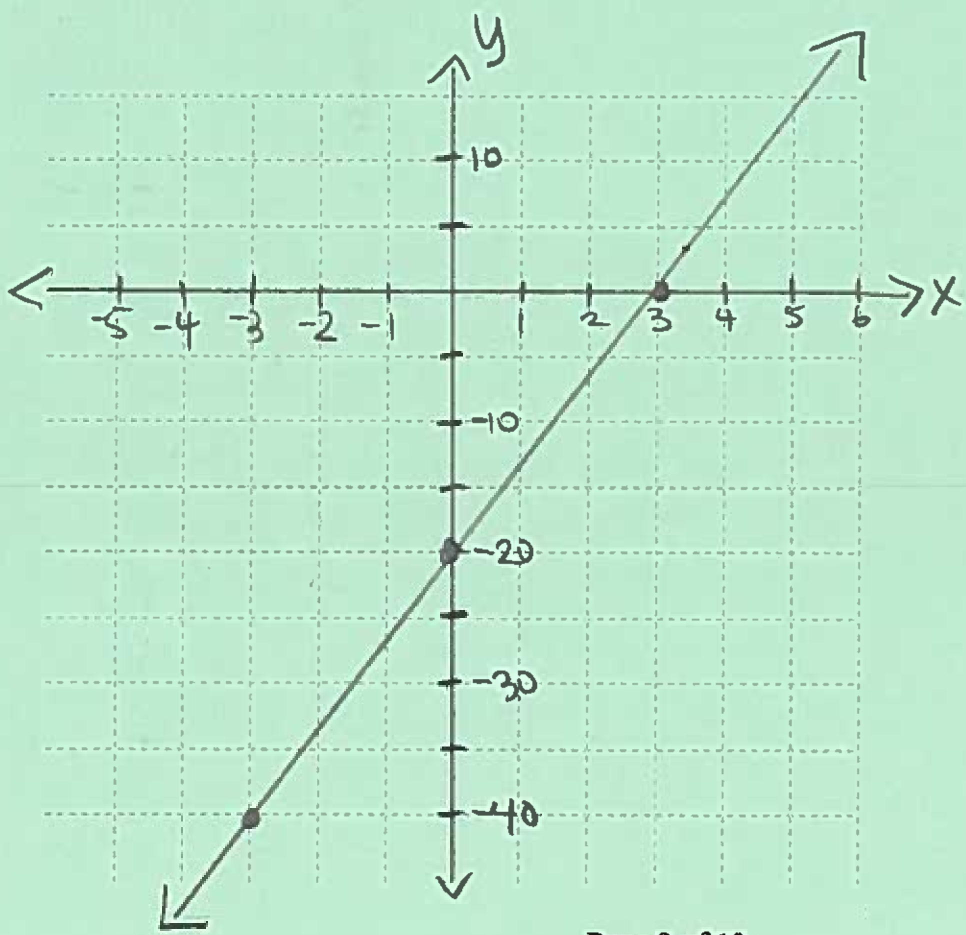
$-60 - 3y + 60 = 60 + 60$

$-3y = 120$

$\frac{-3y}{-3} = \frac{120}{-3}$

$y = -40$

$(-3, -40)$



x-values

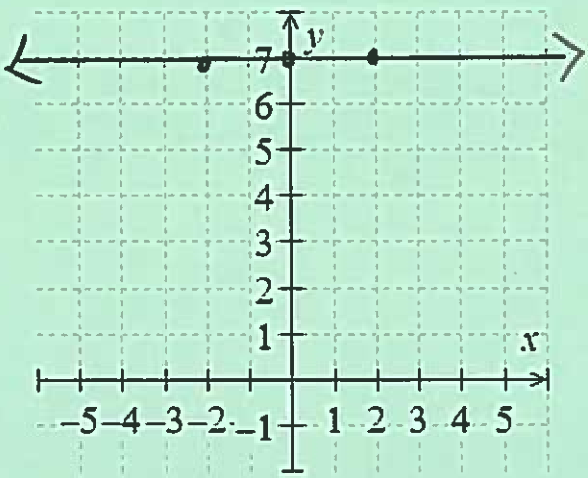
-3, 0, 3

y-values

-40, -20, 0

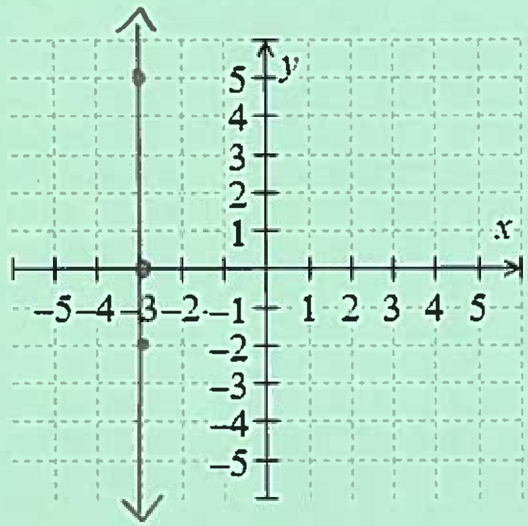
Graph $y = 7$.

x	y
-2	7
0	7
2	7



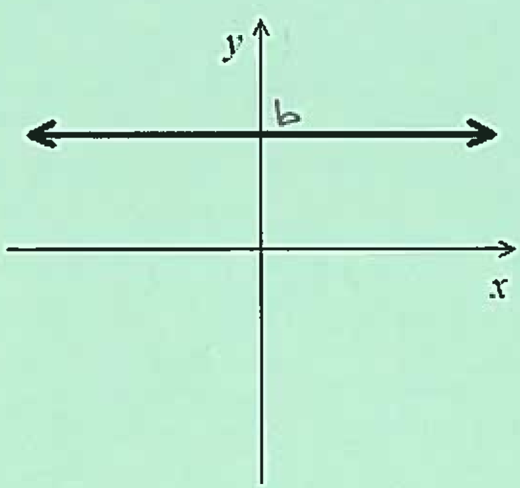
Graph $x = -3$.

x	y
-3	-2
-3	0
-3	5



Horizontal and Vertical Lines

The graph of $y = b$ is a horizontal line.
The y-intercept is $(0, b)$.



The graph of $x = a$ is a vertical line.
The x-intercept is $(a, 0)$.