Example 3:

→ No equal sign

a) In chapter 1, we mostly focused on expressions. What is an expression?

An expression contains sums, differences, products, and/or quotient of variables and constants. (example: $4x+2$)

b) In chapter 2, we've mostly focused on equations. What is an equation?

An equation is a statement that two expressions are equal to one another. (example: $4x+2=y$)

c) How do ratios and proportions relate to expressions and equations?

A ratio is an expression.

A proportion is an equation.

Example 4:

a) What is cross-multiplication?

When solving a proportion we multiply the denominator on the left by the numerator on the right and the numerator on the left by the denominator on the right which brings to mind an X across the equal sign.

b) What is cross-multiplication simply another name for?

the multiplication property of equality

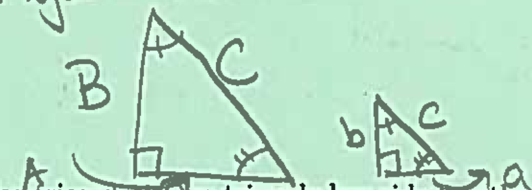
c) Why do many instructors not like cross-multiplication?

Students often try to cross-multiply when the equation is not a proportion and this doesn't work.

Example 5:

a) What are similar triangles?

The angles have the same degree measurement



$\frac{a}{b} = \frac{A}{B}$ $\frac{b}{c} = \frac{B}{C}$

$\frac{a}{c} = \frac{A}{C}$

b) You have two similar triangles. One triangle has side lengths of 4in., 7in., and 9in. The second triangle's longest side length is 12ft. What are the lengths of the other two sides of the second triangle?

Let A represent the length of the shortest side (in inches) of the big triangle and B represent the length of the other unknown side (in inches)

$\frac{4}{9} = \frac{A}{144}$

$144\left(\frac{4}{9}\right) = \cancel{144}\left(\frac{A}{\cancel{144}}\right)$

$\frac{16 \cdot \cancel{9} \cdot 4}{\cancel{9}} = A$

$64 = A$

$\frac{7}{4} = \frac{B}{64}$

$64\left(\frac{7}{4}\right) = \cancel{64}\left(\frac{B}{\cancel{64}}\right)$

$16(7) = B$

$112 = B$

$64 \text{ in} = 5\frac{1}{3} \text{ ft}$

$112 \text{ in} = 9\frac{1}{3} \text{ ft}$

The lengths of the other two sides of the triangle are $5\frac{1}{3}$ ft and $9\frac{1}{3}$ ft.

Example 6:

To estimate how many people have no health insurance in a small city of 50,000 people, a poll of 250 people was taken. 39 people polled had no insurance. How many people in the city can we expect to not have insurance?

Let P represent the number of people in the small city without health insurance.

$\frac{\text{no health insurance}}{\text{total}}$

$\frac{39}{250} = \frac{P}{50000}$

$50000\left(\frac{39}{250}\right) = \cancel{50000}\left(\frac{P}{\cancel{50000}}\right)$

$7800 = P$

We can expect 7800 people to not have health insurance

Example 7:

The owner of a $65,000 house pays $825 in property taxes. Assuming the same tax rate, determine how much tax the owner of a $150,000 house would have to pay.

Let T represent the amount of property tax (in $) the Of a $150,000 house would pay.

$$\frac{\text{property tax}}{\text{house value}} \quad \frac{825}{65000} = \frac{T}{150000}$$

$$150000\left(\frac{825}{65000}\right) = \cancel{150000}\left(\frac{T}{\cancel{150000}}\right)$$

$$1903.85 \approx T$$

The owner of the $150,000 would pay ~~about~~ $1903.85 in property taxes

Example 8:

Scientists needed to figure out how many northern pike (a type of fish) live in Little Lake Hubert, scientists caught, tagged, and released 121 northern pike in the lake. A week later, the scientists returned and caught 50 pike, 22 of which had tags. Approximately how many northern pike are in the lake?

Let f represent the number of northern pike (fish) in the lake.

$$\frac{\text{tagged}}{\text{total}} \quad \frac{22}{50} = \frac{121}{f}$$

$$22f = 121(50)$$

$$22f = 6050$$

$$\frac{22f}{22} = \frac{6050}{22}$$

$$f = 275$$

There are approximately 275 northern pike in the lake.

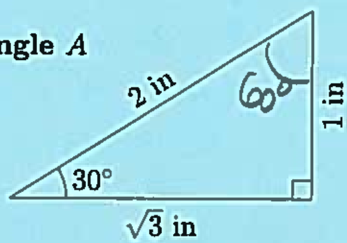
Section 2.8 supplement

An Introduction to Right Triangle Trigonometry

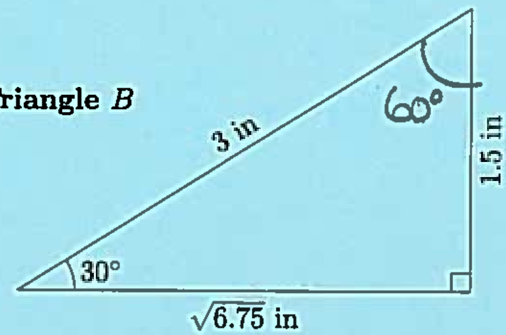
Definitions and Vocabulary:

Consider the following two right triangles

Triangle A



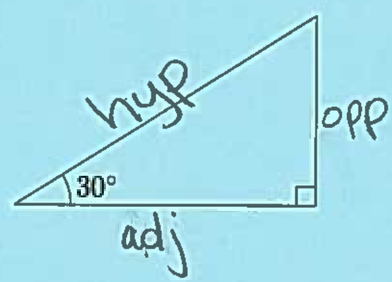
Triangle B



a) Are these similar triangles? Why or why not? yes

The angles have the same measures.

b) From the perspective of the $30°$ angle, identify the adjacent side (adj), opposite side (opp), and the hypotenuse (hyp).

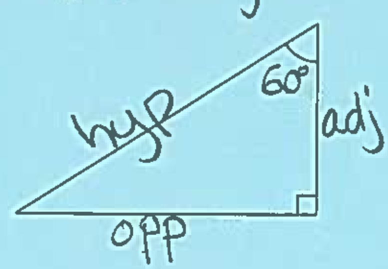


c) What is the measure of the top angle? How do you know? Label that angle in the triangle below.

60°

The sum of the measures of the angles in a triangle is 180°

d) From the perspective ~~of~~ the $60°$ angle, identify the adjacent side (adj), opposite side (opp), and the hypotenuse (hyp).



measure smaller than $90°$

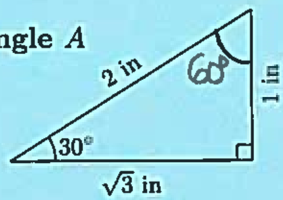
Definitions and Vocabulary:

The **sine** of an acute angle of a right triangle is the ratio of $\dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$.
This is abbreviated as **sin**.

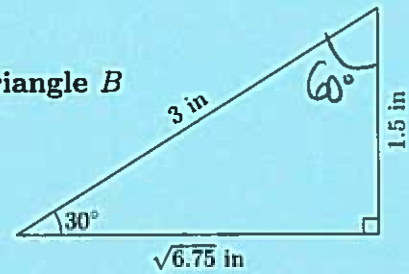
Example 1:

Consider the following two right triangles

Triangle A



Triangle B



a) Find the sine of the 30° angle of each triangle.

i) Triangle A:

$$\sin(30°) = \frac{opp}{hyp} = \frac{1}{2}$$

ii) Triangle B:

$$\sin(30°) = \frac{opp}{hyp} = \frac{1.5}{3} = \frac{1.5}{3} \cdot \frac{10}{10} = \frac{15}{30} = \frac{15 \cdot 1}{15 \cdot 2} = \frac{1}{2}$$

b) Find the sine of the 60° angle of each triangle.

i) Triangle A:

$$\sin(60°) = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$$

≈ 0.866

approximate to compare

ii) Triangle B:

$$\sin(60°) = \frac{opp}{hyp} = \frac{\sqrt{6.75}}{3}$$

≈ 0.866

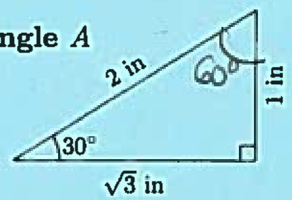
Definitions and Vocabulary:

The **cosine** of an acute angle of a right triangle is the ratio of $\dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$.
This is abbreviated as **cos**.

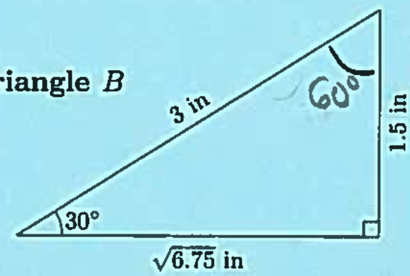
Example 2:

Consider the following two right triangles

Triangle A



Triangle B



a) Find the cosine of the 30° angle of each triangle.

i) Triangle A:

$$\cos(30°) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

ii) Triangle B:

$$\cos(30°) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{6.75}}{3}$$

b) Find the cosine of the 60° angle of each triangle.

i) Triangle A:

$$\cos(60°) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

ii) Triangle B:

$$\cos(60°) = \frac{\text{adj}}{\text{hyp}} = \frac{1.5}{3} = \frac{1}{2}$$

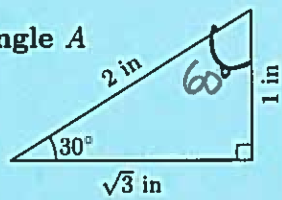
Definitions and Vocabulary:

The **tangent** of an acute angle of a right triangle is the ratio of $\frac{\text{length of opposite side}}{\text{length of adjacent side}}$.
This is abbreviated as **tan**.

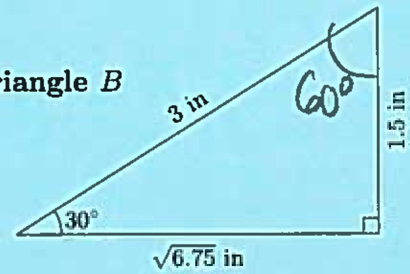
Example 3:

Consider the following two right triangles

Triangle A



Triangle B



a) Find the tangent of the 30° angle of each triangle.

i) Triangle A:

$$\tan(30^\circ) = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{1}{\sqrt{3}}$$
approximate to compare
$$\approx 0.577$$

ii) Triangle B:

$$\tan(30^\circ) = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{1.5}{\sqrt{6.75}}$$
$$\approx 0.577$$

b) Find the tangent of the 60° angle of each triangle.

i) Triangle A:

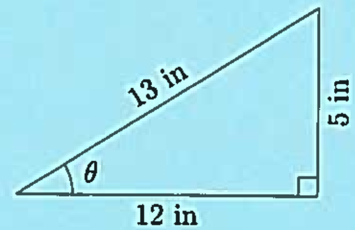
$$\tan(60^\circ) = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{\sqrt{3}}{1}$$
$$= \sqrt{3}$$
approximate to compare
$$\approx 1.732$$

ii) Triangle B:

$$\tan(60^\circ) = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{\sqrt{6.75}}{1.5}$$
$$\approx 1.732$$

Example 4:

Given below is a right triangle. Find the following ratios with respect to angle θ.
(Note: θ (theta) is a Greek letter than is commonly used as a variable to represent the measure of an angle.)



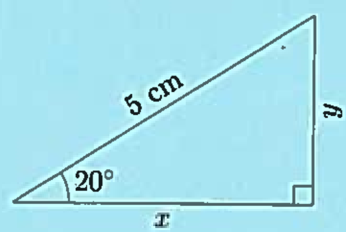
a) $\sin(\theta) = \frac{opp}{hyp} = \frac{5}{13}$

b) $\cos(\theta) = \frac{adj}{hyp} = \frac{12}{13}$

c) $\tan(\theta) = \frac{opp}{adj} = \frac{5}{12}$

Example 5:

Given below is a right triangle.



a) Find x accurate to four decimal places.

$\cos(20°) = \frac{adj}{hyp}$

$\cos(20°) = \frac{x}{5}$

$5\cos(20°) = \cancel{5}\left(\frac{x}{\cancel{5}}\right)$

$4.6985 \approx x$

b) Find y accurate to four decimal places.

$\sin(20°) = \frac{opp}{hyp}$

$\sin(20°) = \frac{y}{5}$

$5\sin(20°) = y$

$1.7101 \approx y$

Definitions and Vocabulary:
Let θ be an acute angle of a right triangle. We have the three following trigonometric functions:

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}} \qquad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

We also have the three reciprocal trigonometric functions cosecant, secant, and cotangent. Abbreviated as csc, sec, cot, these are defined as:

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} \qquad \sec(\theta) = \frac{\text{hyp}}{\text{adj}} \qquad \cot(\theta) = \frac{\text{adj}}{\text{opp}}$$

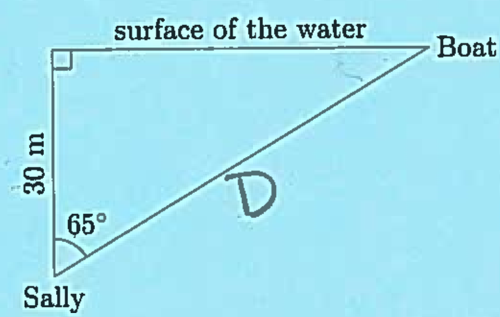
A common nemonic for remembering the first three is SOH-CAH-TOA, representing the fact that...

Sine is **O**pposite over **H**ypotenuse

Cosine is **A**djacent over **H**ypotenuse

Tangent is **O**pposite over **A**djacent

Example 6:
Sally is scuba diving. When she is 30 meters below the surface of the water, she looks up to see her boat and notices that there is a 65 degree angle between the water directly above her and her boat.



How far is Sally from her boat? Let D represent Sally's distance from the boat (in meters).

$$\cos(65°) = \frac{30}{D}$$

$$D \cdot \cos(65°) = \frac{30}{D} \cdot D$$

$$D\cos(65°) = 30$$

$$\frac{D\cos(65°)}{\cos(65°)} = \frac{30}{\cos(65°)}.$$

$$D = \frac{30}{\cos(65°)}$$

$$D \approx 70.99$$

Sally is about 70.99 m from her boat.