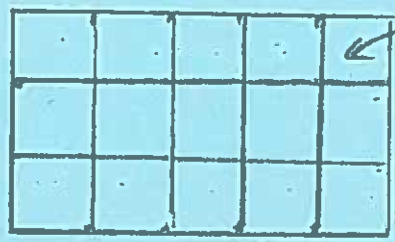
MTH 60 Lecture Notes

Section 2.6: Problem Solving in Geometry

(1) **Rectangles: Formulas for Area and Perimeter** *(To Be Memorized)*

(a) Draw a rectangle that is 3 units high and 5 units wide.



Each is 1 square unit

(b) Draw a grid on your rectangle above that shows the 3-unit height and the 5-unit width.

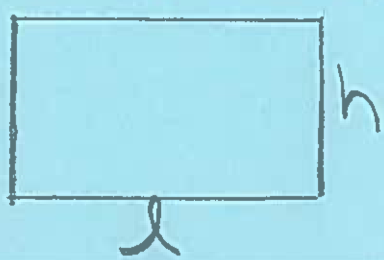
(c) What is the area of the rectangle?

15 square units

(d) What is the perimeter of the rectangle? $3+5+3+5$

16 units

(e) Draw a generic rectangle and label the height h and the length l.



(f) What is the area of the generic rectangle?

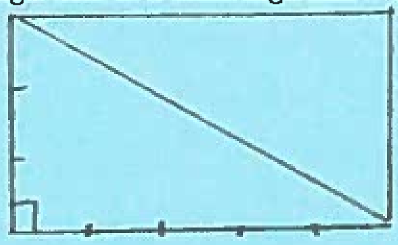
area $= lh$

(g) What is the perimeter of the generic rectangle?

perimeter $= 2l + 2h$

(2) **Triangle: Formula for Area and Angle Information** *(To Be Memorized)*

(a) Draw a triangle that is 3 units high and 5 units wide.

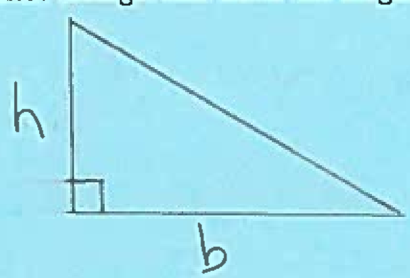


(b) Superimpose upon the triangle a rectangle that has the exact same base and same height.

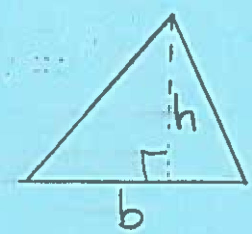
(c) What is the area of the ~~rectangle~~ triangle? $\frac{1}{2}(15)$

7.5 square units

(d) Draw a generic triangle and label the height h and the length of the base b.



or



(e) What is the area of the generic triangle?

$\text{area} = \frac{1}{2}hb$

(f) If you add the angle measures of all the angles of any triangle, what does it add up to make?

180°

(3) **Relationships Between Angles** *(To Be Memorized)*

(a) What are complementary angles?

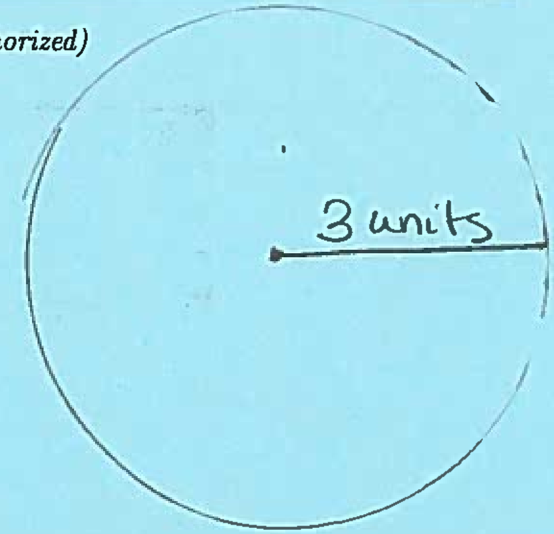
The sum of two complementary angles is 90°.

(b) What are supplementary angles?

The sum of two supplementary angles is 180°.

(4) **Circles: Formulas for Area and Perimeter** *(To Be Memorized)*

(a) Draw a circle that has a radius of 3 units.



(b) What is the diameter of this circle?

6 units

(c) What is the area of the circle?

$A = \pi r^2$ $\quad A = \pi \cdot 3^2$

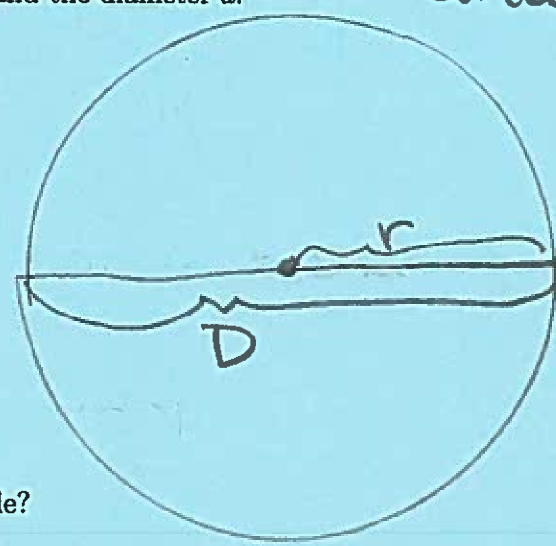
9π square units

(d) What is the circumference (perimeter) of the circle? 6π units

$C = \pi D$ where D is the diameter of the circle

$= 2\pi r$

(e) Draw a generic circle and label the radius r and the diameter d.



(f) What is the area of the generic circle?

$A = \pi r^2$

(g) What is the circumference of the generic circle?

$C = \pi D$

$= 2\pi r$

(h) True or False: $\pi = 3.14$.

False! $\quad \pi \approx 3.14 \quad \pi > 3.14$

(i) True or False: You should approximate π in a math class.

False! Unless you are asked to approximate

(5) A triangular flag is 4 feet high and 1.5 feet wide. What is the area of the flag?

$$\frac{1}{2}(4)(1.5) = 2(1.5)$$
$$= 3$$

The area of the flag is $3\ ft^2$.

(6) A rectangular swimming pool is 75 feet long and 45 feet wide. What is the surface area of the pool?

$A = lh$
$A = lw$

$(75ft)(45ft) = 3375 ft^2$

The surface area of the pool is $3375 ft^2$.

(7) A rectangular swimming pool is 75 feet long and 45 feet wide. What is the perimeter of the pool?

$P = 2l + 2w$

$$2(75ft) + 2(45ft) = 150ft + 90ft$$
$$= 240ft$$

The perimeter of the pool is 240ft.

(8) One angle is 34° more than that of its complement. What are the measures of the angles?
(Approach this as a formal word problem.)

Let A represent the measure of the angle (in degrees)

The complement has measure A + 34 degrees

$$A + A + 34 = 90$$
$$2A + 34 = 90$$
$$2A + 34 - 34 = 90 - 34$$
$$2A = 56$$
$$\frac{2A}{2} = \frac{56}{2}$$

$\to A = 28$

$$A + 34 = 28 + 34$$
$$= 62$$

check: $28 + 62 = 90$

The measures of the angles are 28° and 62°.

(9) When working with lines and other one-dimensional distances, what are examples of appropriate units?

feet miles
km meters
yards

(10) When working with the perimeter of two-dimensional shapes, what are examples of appropriate units?

feet miles
km meters
yards

(11) When working with the area of two-dimensional shapes, what are examples of appropriate units?

ft^2 yd^2
km^2 mi^2
m^2

(12) Why are the areas of two-dimensional shapes always "square" units?

We multiply the units to compute area giving us square units.

(13) If we move to three-dimensional shapes, we can discuss the surface area and the volume of the shape.

(a) What is meant by the surface area of a three-dimensional shape?

Add the areas of all the sides

(b) What is meant by the volume of a three-dimensional shape?

the inside

how much it will hold

(14) When working with the volume of three-dimensional shapes, what are examples of appropriate units?

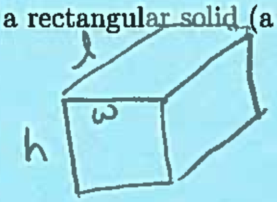
Liters gallons mi^3
ft^3 km^3 yd^3 m^3 in^3

(15) Why are the volumes of three-dimensional shapes always "cube" units?

There is a third dimension; we are multiplying the unit together 3 times.

(16) **Rectangular Solids (boxes): Formula for Volume** *(To Be Memorized)*

(a) Draw a rectangular solid (a box) that is h units high, w units wide, and l units long.



(b) What is the volume of the rectangular solid?

$V = lwh$

(c) Explain why this formula makes sense. multiplying just lw gives the area of the top of the box. Multiplying lw by h gives the third dimension.

(17) Right Circular Cylinder ~~**Rectangular Solids (boxes)**~~**: Formula for Volume** *(To Be Memorized)*

(a) Draw a right circular cylinder (a can of beans) that is h units high and has a radius of r units.

(b) What is the volume of the circular cylinder?

$V = \pi r^2 h$

(c) Explain why this formula makes sense. Find the area of the circle first, πr^2, and then multiply by h to give the third dimension.

(18) Why have I not given you any formulas for squares (two-dimensional) or cubes (three-dimensional)?

A square is a special case of a rectangle and a cube is a special case of a rectangular solid so we can use the same formulas.

isosceles triangle

(19) The first angle of a triangle has the same measure as the second angle. The third angle measures 42 degrees less than the first angle. What are the measures of the three angles?

Let A represent the measure of the triangle's first angle (in degrees).

The sum of all three angle measures is 180°.

measures of 1st ∠ + 2nd ∠ + 3rd ∠ = 180°

$$A + A + (A-42) = 180$$
$$A + A + A - 42 = 180$$
$$3A - 42 = 180$$
$$3A - 42 + 42 = 180 + 42$$
$$3A = 222$$

(20) ~~The measure of one complementary angle is three degrees less than twice the measure of the second angle. What are the measures of the two angles?~~

$$\frac{3A}{3} = \frac{222}{3}$$
$$A = 74$$

3rd angle

$$74 - 42 = 32$$

The measures of the three angles of the triangle are 32°, 74°, and 74°.

(21) A rectangle has a width of 23 inches and a perimeter of 65 inches. What is the rectangle's length?

Let L represent the length of the rectangle (in inches)

width = 23 in.

perimeter = 65 in.

$$P = 2l + 2w$$
$$65 = 2L + 2(23)$$
$$65 = 2L + 46$$

(22) ~~A rectangle has a length of 14 centimeters and an area of 133 square centimeters. What is the rectangle's width?~~

$$65 - 46 = 2L + 46 - 46$$
$$19 = 2L$$
$$\frac{19}{2} = \frac{2L}{2}$$
$$9.5 = L$$

The length of the rectangle is 9.5 inches.

(23) **Formulas and Knowledge To Be Memorized**

(a) **Rectangles**

- Area: $A = lw$
- Perimeter: $P = 2l + 2w$

(b) **Triangles**

- Area: $A = \frac{1}{2}bh$
- Angles: Always add up to 180°

(c) **Angles**

- Complementary: two angles whose measures add up to 90°
- Supplementary: two angles whose measures add up to 180°

(d) **Circles**

- Area: $A = \pi r^2$
- Circumference: $2\pi r$

(e) **Rectangular Solids**

- Volume: $V = lwh$

(f) **Triangles**

- Volume: $V = \pi r^2 h$

(g) **Units**

- One-Dimensional: feet, meters, inches, centimeters, etc.
- Two-Dimensional: square feet, square meters, square inches, square centimeters, etc.
- Three-Dimensional: cube feet, cube meters, cube inches, cube centimeters, etc.

Section 2.8

Ratios and Proportions

Example 1:

Solve.

a) $\frac{x}{3}+\frac{1}{2}=\frac{x}{2}-\frac{5}{6}$.

$$6\left(\frac{x}{3}+\frac{1}{2}\right)=6\left(\frac{x}{2}-\frac{5}{6}\right)$$

$$\frac{6x}{3}+\frac{6}{2}=\frac{6x}{2}-\frac{30}{6}$$

$$2x+3=3x-5$$

$$2x+3-3x=3x-5-3x$$

$$-x+3=-5$$

$$-x+3-3=-5-3$$

$$-x=-8$$

$$\frac{-x}{-1}=\frac{-8}{-1}$$

$$x=8$$

The solution is 8.

b) $\frac{x}{5}=\frac{8}{10}$

$$\cancel{5}\left(\frac{x}{\cancel{5}}\right)=5\left(\frac{8}{10}\right)$$

$$x=\frac{40}{10}$$

$$x=4$$

The solution is 4.

c) $\frac{45}{x}=\frac{5}{8}$

$$\cancel{x}\left(\frac{45}{\cancel{x}}\right)=x\left(\frac{5}{8}\right)$$

$$45=\frac{5}{8}x$$

$$\frac{8}{5}(45)=\frac{\cancel{8}}{\cancel{5}}\left(\frac{\cancel{5}}{\cancel{8}}x\right)$$

$$8(9)=x$$

$$72=x$$

The solution is 72.

d) $-\frac{3}{8x}=\frac{1}{40}$

$$\left(-\frac{3}{8x}\right)x=\left(\frac{1}{40}\right)x$$

$$-\frac{3}{8}=\frac{1}{40}x$$

$$\frac{40}{1}\left(-\frac{3}{8}\right)=\frac{\cancel{40}}{1}\left(\frac{1}{\cancel{40}}x\right)$$

$$\frac{5\cdot\cancel{8}(-3)}{1\cdot\cancel{8}}=x$$

$$-15=x$$

The solution is -15.

e) $\frac{5}{x-2}=\frac{10}{3}$

$$\frac{5}{\cancel{x-2}}(\cancel{x-2})=\frac{10}{3}(x-2)$$

$$5=\frac{10x-20}{3}$$

$$3(5)=\cancel{3}\left(\frac{10x-20}{\cancel{3}}\right)$$

$$15=10x-20$$

$$15+20=10x-20+20$$

$$35=10x$$

$$\frac{35}{10}=\frac{10x}{10}$$

$$\frac{\cancel{5}\cdot7}{\cancel{5}\cdot2}=x$$

$$\frac{7}{2}=x$$

The solution is $\frac{7}{2}$.

Definitions and Vocabulary:

a) What is a ratio? A comparison

b) Give three examples of ratios used in your life each day.

rates

cooking $\frac{\text{flour}}{\text{water}}$ $\frac{\text{7 days}}{\text{1 week}}$ $\frac{\text{mi}}{\text{gal}}$ gardening $\frac{\text{fertilizer}}{\text{soil}}$ tablespoons of $\frac{\text{coffee}}{\text{water}}$

c) A 4th of July Fun Size bag of M&M's contained 8 blue, 5 red, and 11 white M&M's.

(a) Find the ratio of red M&M's to blue M&M's.

5 to 8

5:8

$\frac{5}{8}$

(b) Find the ratio of blue M&M's to white M&M's.

$\frac{8}{11}$

(c) Find the ratio of white M&M's to total number of M&M's.

$\frac{11}{24}$

(d) Find the ratio of total number of M&M's to red M&M's.

$\frac{24}{5}$

(e) Find the percentage of M&M's in the bag that were red.

$\frac{5}{24} \approx 20.8\%$

(f) Find the percentage of M&M's in the bag that were white.

$\frac{11}{24} \approx 45.8\%$

Definitions and Vocabulary:

a) What is a proportion? Compares two ratios

one ratio = another ratio

b) Look back at the previous page. Which of those problems were proportions?

b, c, d, and e

Example 2:

Solve.

a) $\frac{2}{y-5} = \frac{3}{y+6}$

$$\frac{2}{\cancel{y-5}}\cancel{(y-5)}(y+6) = \frac{3}{\cancel{y+6}}(y-5)\cancel{(y+6)}$$

$$2(y+6) = 3(y-5)$$

$$2y+12 = 3y-15$$

$$2y+12-2y = 3y-15-2y$$

$$12 = y-15$$

$$12+15 = y-15+15$$

$$27 = y$$

The solution is 27.

Check: $y=27$

$$\frac{2}{y-5} = \frac{3}{y+6}$$

$$\frac{2}{27-5} \stackrel{?}{=} \frac{3}{27+6}$$

$$\frac{2}{22} \stackrel{?}{=} \frac{3}{33}$$

$$\frac{\cancel{2}\cdot 1}{\cancel{2}\cdot 11} \stackrel{?}{=} \frac{\cancel{3}\cdot 1}{\cancel{3}\cdot 11}$$

$$\frac{1}{11} = \frac{1}{11} \checkmark$$

b) $\frac{y+10}{y-2} = \frac{10}{4}$

$$(y+10)4 = 10(y-2)$$

$$4y+40 = 10y-20$$

$$4y+40-4y = 10y-20-4y$$

$$40 = 6y-20$$

$$40+20 = 6y-20+20$$

$$60 = 6y$$

$$\frac{60}{6} = \frac{6y}{6}$$

$$10 = y$$

The solution is 10.

Example 3:

→ No equal sign

a) In chapter 1, we mostly focused on expressions. What is an expression?

An expression contains sums, differences, products, and/or quotient of variables and constants. (example: $4x+2$)

b) In chapter 2, we've mostly focused on equations. What is an equation?

An equation is a statement that two expressions are equal to one another. (example: $4x+2=y$)

c) How do ratios and proportions relate to expressions and equations?

A ratio is an expression.

A proportion is an equation.

Example 4:

a) What is cross-multiplication?

b) What is cross-multiplication simply another name for?

c) Why do many instructors not like cross-multiplication?