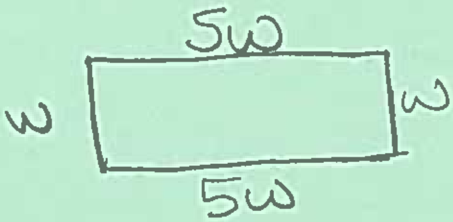
MTH 60

Section 2.5 An Introduction to Problem Solving (continued)

Example: A rectangular field is five times as long as it is wide. If the perimeter of the field is 288 yards, what are the field's dimensions?



Let w represent the width of the field (in yards)

Then the length is 5w.

Perimeter = 288 yards

$$P = 2L + 2W$$

$$288 = 2(5w) + 2w$$

$$288 = 10w + 2w$$

$$288 = 12w$$

$$\frac{288}{12} = \frac{12w}{12}$$

$$24 = w$$

$$\begin{array}{r} 24 \\ 12 \overline{)288} \\ \underline{-24} \\ 48 \\ \underline{48} \\ 0 \end{array} \qquad \begin{array}{r} {}^{2}24 \\ \underline{\times 5} \\ 120 \end{array}$$

$$5w = 5(24)$$
$$= 120$$

The field's dimensions are 24 yd x 120 yd.

$0.09L$

Example: This year's salary, $42,074, is a 9% increase over last year's salary. What was last year's salary?

Let L represent last year's salary (in $).

last year's salary + increase amount (in $) = this year's salary

$$L + 0.09L = 42074$$

$$1.09L = 42074$$

check:

$$\frac{1.09L}{1.09} = \frac{42074}{1.09}$$

$38600 + 0.09(38600) = 4207$

$$L = 38600$$

Last year's salary was $38600.

repair bill − parts = labor

Example: A repair bill on a sailboat came to \$1603, including \$532 for parts and the remainder for labor. If the cost of labor is \$63 per hour, how many hours of labor did it take to repair the sailboat?

Let t represent the time (in hours) it took to repair the sailboat.

$$\underset{\text{bill}}{\text{Total repair}} = \overset{\text{cost of}}{\text{parts}} + \overset{\text{total cost of}}{\text{labor}}$$

$$1603 = 532 + 63t$$

$$1603 - 532 = 532 + 63t - 532$$

$$1071 = 63t$$

$$\frac{1071}{63} = \frac{63t}{63}$$

$$17 = t$$

$$\begin{array}{r} 63 \\ \times\ 7 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 1603 \\ -\ 532 \\ \hline 1071 \end{array}$$

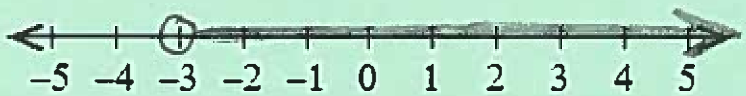
$$\begin{array}{r} 17 \\ 63\,\overline{)\,1071} \\ -63 \\ \hline 441 \\ -441 \\ \hline 0 \end{array}$$

It took 17 hours to repair the sailboat.

Section 2.7 Solving Linear Inequalities

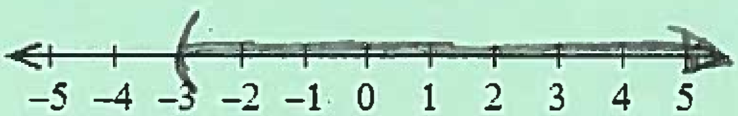
A **linear inequality in one variable** $ax + b \le c$ where a, b, and c are real numbers and the symbol between $ax + b$ and c can be $\le$ (is less than or equal to), $<$ (is less than), $\ge$ (is greater than or equal to), or $>$ (is greater than).

Example: Graph the solution of each inequality.
a. $x > -3$ **b.** $x \le 2$ **c.** $-4 < x \le 3$

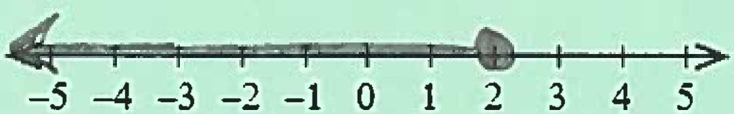


$x > -3$

O or (indicate -3 is not included

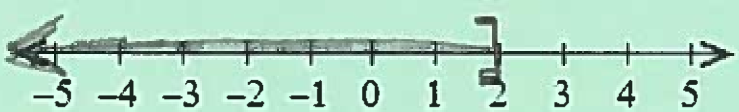


$x > -3$

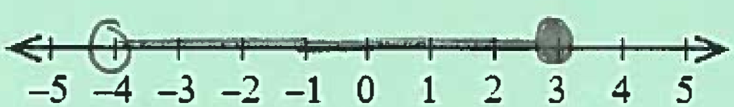


$x \le 2$

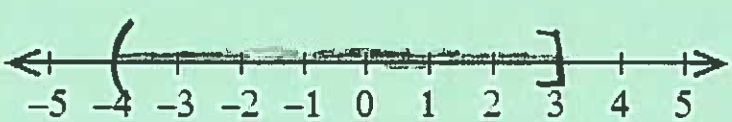
● or] indicate 2 is included



$x \le 2$



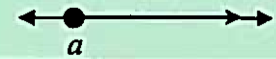
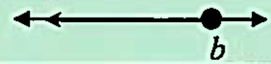
$-4 < x \le 3$



$-4 < x \le 3$

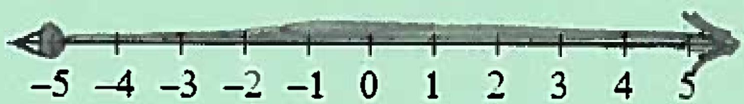
The round parentheses or the open circle mean the endpoint is not included. The square bracket or the filled in circle mean the endpoint **is** included.

such that

Table 1: Solution Sets of Inequalities			
Let a and b represent real numbers.			
Inequality	Interval Notation	Set-Builder Notation	Graph
$x > a$	(a, ∞)	$\{x \mid x > a\}$	
$x \geq a$	$[a, \infty)$	$\{x \mid x \geq a\}$	
$x < b$	$(-\infty, b)$	$\{x \mid x < b\}$	
$x \leq b$	$(-\infty, b]$	$\{x \mid x \leq b\}$	

Example: Express the solution set of each inequality in interval notation and graph the interval.
a. $x \geq -5$ **b.** $x < 4$

a. $[-5, \infty)$



b. $(-\infty, 4)$

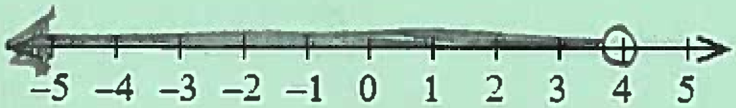


Table 2: Properties of Inequalities		
Property	The Property in Words	Example
The Addition Property of Inequality If $a < b$, then $a + c < b + c$ If $a < b$, then $a - c < b - c$	If the same quantity is added to or subtracted from both sides of an inequality, the resulting inequality is equivalent to the original one.	$3x - 5 < 7$ $3x - 5 + 5 < 7 + 5$ $3x < 12$
The Positive Multiplication Property of Inequality If $a < b$ and c is positive, then $ac < bc$ If $a < b$ and c is positive, then $\frac{a}{c} < \frac{b}{c}$	If we multiply or divide both sides of an inequality by the same positive quantity, the resulting inequality is equivalent to the original one.	$3x < 12$ $\frac{3x}{3} < \frac{12}{3}$ $x < 4$
The Negative Multiplication Property of Inequality If $a < b$ and c is negative, then $ac > bc$ If $a < b$ and c is negative, then $\frac{a}{c} > \frac{b}{c}$	If we multiply or divide both sides of an inequality by the same negative quantity **and reverse the direction of the inequality symbol**, the resulting inequality is equivalent to the original one.	$-5x < 20$ $\frac{-5x}{-5} > \frac{20}{-5}$ $x > -4$

$2 < 7$

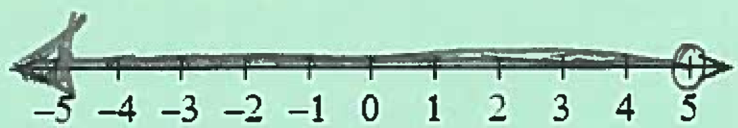
$2(-1) > 7(-1)$

$-2 > -7$

Example: Solve $2x-7<3$ and graph the solution set on a number line. Write the solution set in interval notation.

$$2x-7<3$$
$$2x-7+7<3+7$$
$$2x<10$$
$$\frac{2x}{2}<\frac{10}{2}$$
$$x<5$$

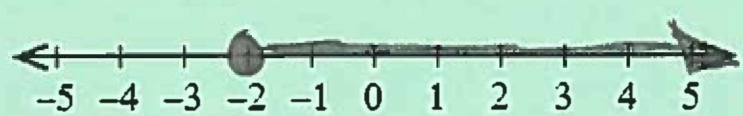
$$(-\infty,5)$$



Example: Solve $3-5x\le 13$ and graph the solution set on a number line. Write the solution set in interval notation.

$$3-5x\le 13$$
$$3-5x-3\le 13-3$$
$$-5x\le 10$$
$$\frac{-5x}{-5}\ge\frac{10}{-5}$$
$$x\ge -2$$

$$[-2,\infty)$$



Example: Solve $4(x+1)+2 \le 3x+6$ and graph the solution set on a number line. Write the solution set in interval notation.

$4(x+1)+2 \le 3x+6$

$4x+4+2 \le 3x+6$

$4x+6 \le 3x+6$

$4x+6-3x \le 3x+6-3x$

$x+6 \le 6$

$x+6-6 \le 6-6$

$x \le 0$

$(-\infty, 0]$

Example: Solve $x+2 < x+5$ and write the solution set in interval notation.

$x+2 < x+5$

$x+2-x < x+5-x$

$2 < 5$ The variables are eliminated and we end up with a true statement.

Every real number is a solution.

$(-\infty, \infty)$

Note: If the variables are eliminated in an inequality and we end up with a false statement, this means there are no solutions.

{ } means the empty set

Recognizing Inequalities with No Solution or Infinitely Many Solutions
If you attempt to solve an inequality with no solution or one that is true for every real number, you will eliminate the variable.

- An inequality with no solution results in a false statement, such as $0>1$. The solution set is $\varnothing$, the empty set.
- An inequality that is true for every real number results in a true statement, such as $0<1$. The solution set is $(-\infty,\infty)$, which can be written as $\{x \mid x \text{ is a real number}\}$.

Example: Solve $5(x+1)>5x+8$.

$$5(x+1) > 5x+8$$
$$5x+5 > 5x+8$$
$$5x+5-5x > 5x+8-5$$
$$5>8 \quad \text{False}$$

There are no solutions

The solution set is $\varnothing$.