

Section 2.3 Solving Linear Equations

Solving a Linear Equation

1. Simplify the algebraic expression on each side.
2. Collect all the variable terms on one side and all the constant terms on the other side.
3. Isolate the variable and solve.
4. Check the proposed solution in the original equation. Use the order of operations when performing this check.
5. Write your answer in a sentence.

Example: Solve and check $3x + 2 - x = 6 + 3x - 8$.

$$3x + 2 - x = 6 + 3x - 8$$

$$2x + 2 = -2 + 3x$$

$$2x + 2 - 2 = -2 + 3x - 2$$

$$2x = -4 + 3x$$

$$2x - 3x = -4 + 3x - 3x$$

$$-x = -4$$

$$\frac{-x}{-1} = \frac{-4}{-1}$$

$$x = 4$$

Check: $x = 4$

$$3x + 2 - x = 6 + 3x - 8$$

$$3(4) + 2 - 4 \stackrel{?}{=} 6 + 3(4) - 8$$

$$12 + 2 - 4 \stackrel{?}{=} 6 + 12 - 8$$

$$14 - 4 \stackrel{?}{=} 18 - 8$$

$$10 = 10$$

The solution is 4.

Example: Solve and check $3(3z+5)-7=89$.

$$\begin{aligned}3(3z+5)-7 &= 89 \\ 9z+15-7 &= 89 \\ 9z+8 &= 89 \\ 9z+8-8 &= 89-8 \\ 9z &= 81 \\ \frac{9z}{9} &= \frac{81}{9} \\ z &= 9\end{aligned}$$

Check: If $z=9$,
then $3(3z+5)-7 = 3(3 \cdot 9+5)-7$
 $= 3(27+5)-7$
 $= 3(32)-7$
 $= 96-7$
 $= 89 \checkmark$

The solution is 9.

scratch

$$\begin{array}{r} 33 \\ \times 5 \\ \hline 165 \end{array} \quad \begin{array}{r} 2 \\ 83 \\ -9 \\ \hline 24 \end{array}$$

Example: Solve and check $5m-4(m+9)=2m-3$.

$$\begin{aligned}5m-4(m+9) &= 2m-3 \\ 5m-4 \cdot m - 4 \cdot 9 &= 2m-3 \\ 5m-4m-36 &= 2m-3 \\ m-36 &= 2m-3 \\ m-36+3 &= 2m-3+3 \\ m-33 &= 2m \\ m-33-m &= 2m-m \\ -33 &= m\end{aligned}$$

Check: $m=-33$

$$\begin{aligned}5m-4(m+9) &= 2m-3 \\ 5(-33)-4(-33+9) &\stackrel{?}{=} 2(-33)-3 \\ -165-4(-24) &\stackrel{?}{=} -66-3 \\ -165+96 &\stackrel{?}{=} -69 \\ -69 &= -69\end{aligned}$$

scratch

$$\begin{array}{r} 15 \\ 165 \\ -96 \\ \hline 69 \end{array}$$

The solution is -33.

Example: Solve and check $-2(k-4) - (3k-2) = -2 - (6k-2)$.

$$-2(k-4) - (3k-2) = -2 - (6k-2)$$

$$-2k + 8 - 3k + 2 = -2 - 6k + 2$$

$$-5k + 10 = -6k$$

$$-5k + 10 + 5k = -6k + 5k$$

$$10 = -k$$

$$\frac{10}{-1} = \frac{-k}{-1}$$

$$-10 = k$$

Check: $k = -10$

$$-2(k-4) - (3k-2) = -2 - (6k-2)$$

$$-2(-10-4) - (3(-10)-2) \stackrel{?}{=} -2 - (6(-10)-2)$$

$$-2(-14) - (-30-2) \stackrel{?}{=} -2 - (-60-2)$$

$$28 - (-32) \stackrel{?}{=} -2 - (-62)$$

$$28 + 32 \stackrel{?}{=} -2 + 62$$

$$60 = 60 \checkmark$$

The solution is -10 .

Use the multiplication property of equality to eliminate the fractions as our first step.

Example: Solve and check $\frac{3y}{4} - \frac{2}{3} = \frac{7}{12}$.

$$\frac{3y}{4} - \frac{2}{3} = \frac{7}{12}$$

$$12\left(\frac{3y}{4} - \frac{2}{3}\right) = 12\left(\frac{7}{12}\right)$$

$$\frac{12 \cdot 3y}{4} - \frac{12 \cdot 2}{3} = 7$$

$$\frac{36y}{4} - \frac{24}{3} = 7$$

$$9y - 8 = 7$$

$$9y - 8 + 8 = 7 + 8$$

$$9y = 15$$

$$\frac{9y}{9} = \frac{15}{9}$$

$$y = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 3}$$

$$y = \frac{5}{3}$$

Check: ^{If} $y = \frac{5}{3}$

$$\text{then } \frac{3y}{4} - \frac{2}{3} = \frac{3}{4}\left(\frac{5}{3}\right) - \frac{2}{3}$$

$$= \frac{15}{12} - \frac{2}{3} \cdot \frac{4}{4}$$

$$= \frac{15}{12} - \frac{8}{12}$$

$$= \frac{7}{12} \checkmark$$

The solution is $\frac{5}{3}$.

Recognizing Inconsistent Equations and Identities

If you attempt to solve an equation with no solution or one that is true for every real number, you will eliminate the variable.

- An inconsistent equation with no solution results in a false statement, like $2 = 5$
- An identity that is true for every real number results in a true statement, like $4 = 4$

Example: Solve $3(p-7) = 3p-21$.

$$3(p-7) = 3p-21$$

$$3p-21 = 3p-21$$

$$3p-21+21 = 3p-21+21$$

$$3p = 3p$$

$$3p-3p = 3p-3p$$

$$0 = 0 \text{ Identity}$$

Every real number is a solution.

As soon as you recognize you have an identity, you can label it as such and write the sentence.



Example: Solve $5r-5-2(r+1) = 3r-9$.

$$5r-5-2(r+1) = 3r-9$$

$$5r-5-2r-2 = 3r-9$$

$$3r-7 = 3r-9$$

$$3r-7-3r = 3r-9-3r$$

$$-7 = -9 \text{ False}$$

Inconsistent equation

There are no solutions.

Section 2.4 Percents

$n\%$ means $\frac{n}{100}$.

To write a percent as a decimal, move the decimal point two places to the left and remove the percent sign.

Example: Express each percent as a decimal. a. 14% b. 167%

a. $14\% = 0.14$

b. $167\% = 1.67$

To write a decimal as a percent, move the decimal point two places to the right and attach a percent sign.

Example: Express 0.73 as a percent.

$$0.73 = 73\%$$

We can use the following formula involving percent.

A is P percent of B

$$A = P \cdot B, \text{ usually written } A = PB$$

Example: What is 12% of 30?

Let A be the number we are looking for.

$$A = 0.12(30) \\ = 3.6$$

$$\begin{array}{r} 30 \\ \times 0.12 \\ \hline 60 \\ 300 \\ \hline 3.60 \end{array}$$

3.6 is 12% of 30.

$$A = PB$$

Example: 7 is 20% of what?
A P B

Let B represent the number I am looking for.

$$7 = 0.20B$$

$$\frac{7}{0.20} = \frac{0.20B}{0.20}$$

$$35 = B$$

$$\begin{array}{r} 2 \overline{) 70} \\ \underline{40} \\ 30 \\ \underline{20} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

7 is 20% of 35.

Example: 1.6 is what percent of 40?

Let P represent the unknown percent
(in decimal form)

$$1.6 = P \cdot 40$$

$$1.6 = 40P$$

$$\frac{1.6}{40} = \frac{40P}{40}$$

$$0.04 = P$$

$$4\% = P$$

$$\begin{array}{r} 40 \overline{) 1.60} \\ \underline{160} \\ 0 \end{array}$$

1.6 is 4% of 40.

Suppose that the local sales tax rate is 7% and you buy a graphing calculator for \$96.

- a. How much tax is due?
- b. What is the calculator's total cost?

\$ is called
the unit

a. What is 7% of 96?

Let T represent the amount of tax due (in \$).

$$\begin{aligned} T &= 0.07(96) \\ &= 6.72 \end{aligned}$$

$$\begin{array}{r} 96 \\ \times 0.07 \\ \hline 6.72 \end{array}$$

The tax due is \$6.72.

b. $96 + 6.72 = 102.72$

The calculator's total cost is \$102.72.