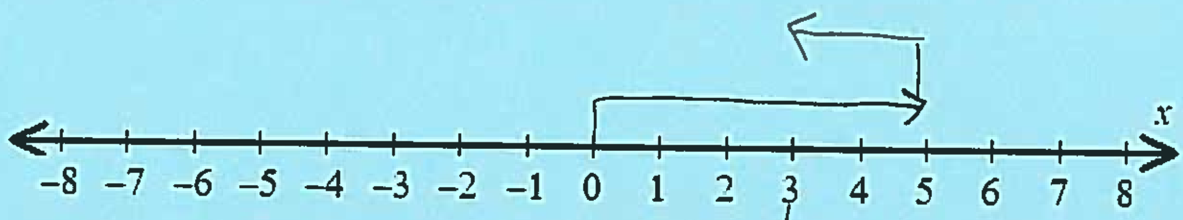
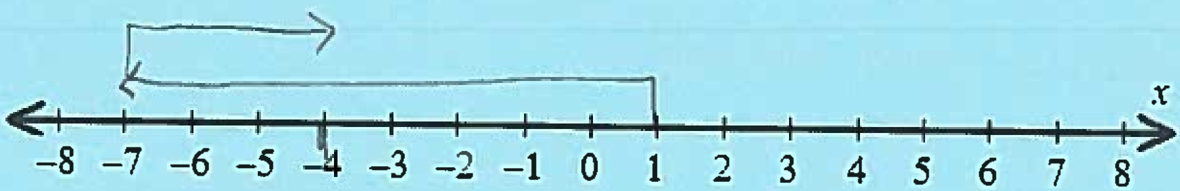
Section 1.5 Addition of Real Numbers

If you won $5 and then lost $2, you would have $3 left. Let's represent this on a number line.



$5+(-2)=3$

If you lost $7 and then won $3, overall you would have lost $4. Let's represent this on a number line.

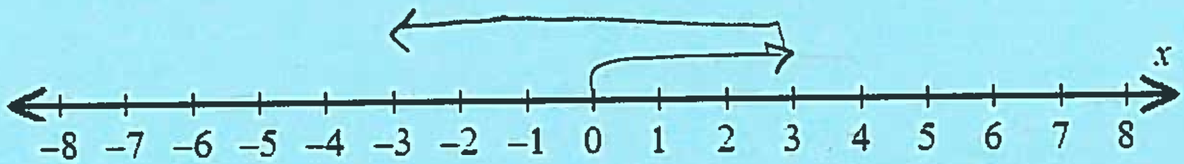


$-7+3=-4$

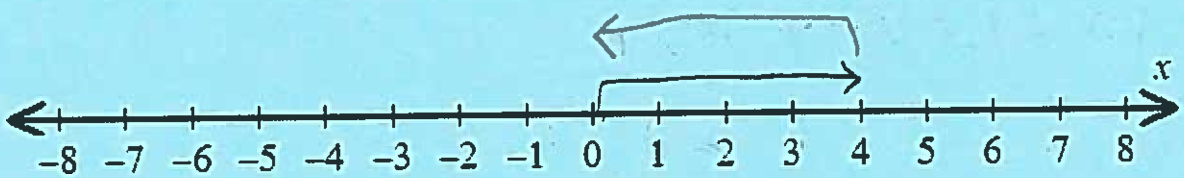
> Using the Number Line to Find a Sum
> Let a and b represent real numbers. To find $a+b$ using a number line,
> 1. Start at a.
> 2a. If b is **positive**, move b units to the **right**.
> b. If b is **negative**, move b units to the **left**.
> c. If b is **0**, **stay** at a.
> 3. The number where we finish on the number line represents the sum of a and b.

Example: Find the sum using a number line.

a. $3+(-6)$ = -3



b. $4+(-4)$ = 0



Numbers that differ only in sign, such as 4 and -4, are called *additive inverses*. **Additive inverses**, also called **opposites**, are pairs of real numbers that are the same number of units from zero on the number line, but are on opposite sides of zero.

> **Identity and Inverse Properties of Addition**
>
> Let a be a real number, a variable, or an algebraic expression.
>
> Identity Property of Addition: $a+0=a$, $0+a=a$ — 0 is the **additive identity**.
>
> Inverse Property of Addition: $a+(-a)=0$, $(-a)+a=0$

Example: Find the sum. **a.** $0+(3m-4)$ **b.** $[-(2a+7)]+(2a+7)$

a. $0+(3m-4)=3m-4$

b. $[-(2a+7)]+(2a+7)=0$

Adding Two Integers That Have the Same (Like) Signs

1. To add two positive integers, add them as usual. The final answer is positive.
2. To add two negative integers, add their absolute values and make the final answer negative.

Example: Add. **a.** $-9+(-11)$ **b.** $-0.3+(-0.5)$ **c.** $-\frac{1}{2}+\left(-\frac{2}{3}\right)$

a. $-9+(-11)=-20$

b. $-0.3+(-0.5)=-0.8$

c. $-\frac{1}{2}+\left(-\frac{2}{3}\right)=-\left(\frac{1}{2}\cdot\frac{3}{3}\right)+\left(-\frac{2}{3}\cdot\frac{2}{2}\right)$

$=-\frac{3}{6}+\left(-\frac{4}{6}\right)$

$=-\frac{7}{6}$

Adding Two Integers That Have the Different (Unlike) Signs

To add a positive integer and a negative integer, subtract the smaller absolute value from the larger absolute value.

1. If the positive integer has the larger absolute value, the final answer is positive.
2. If the negative integer has the larger absolute value, the final answer is negative.

Example: Add. **a.** $-12+3$ **b.** $-\frac{1}{4}+\frac{1}{2}$

a. $-12+3=-9$

b. $-\frac{1}{4}+\frac{1}{2}=-\frac{1}{4}+\frac{1}{2}\cdot\frac{2}{2}$

$=-\frac{1}{4}+\frac{2}{4}$

$=\frac{1}{4}$

Example: Simplify. **a.** $-7x+3x$ **b.** $6y+(-3z)+(-8y)+5z$ **c.** $4(2m+3)+(-11m)$

a. $-7x+3x=-4x$

b. $6y+(-3z)+(-8y)+5z = 6y+(-8y)+(-3z)+5z$ ✱
$= -2y+2z$

c. $4(2m+3)+(-11m) = 4\cdot 2m + 4\cdot 3 + (-11m)$ ✱
$= 8m+12+(-11m)$
$= 8m+(-11m)+12$ ✱
$= -3m+12$

✱ These steps are optional.

Section 1.6 Subtraction of Real Numbers

Definition of Subtraction
For all real numbers a and b, $a-b=a+(-b)$.
In words: To subtract b from a, add the opposite, or additive inverse, of b to a. The result of the subtraction is called the **difference**.

Subtracting Real Numbers
1. Change the subtraction operation to addition.
2. Change the sign of the number being subtracted.
3. Add, using one of the rules for adding numbers with the same sign or different signs.

Example: Subtract. $6-9$

$6-9=6+(-9)$
$=-3$

Example: Subtract. $3\pi-(-8\pi)$

$$3\pi-(-8\pi)=3\pi+8\pi$$
$$=11\pi$$

Example: Subtract. $-\frac{2}{3}-\frac{3}{4}$

$$-\frac{2}{3}-\frac{3}{4}=-\frac{2}{3}+\left(-\frac{3}{4}\right)$$
$$=-\frac{2}{3}\cdot\frac{4}{4}+\left(-\frac{3}{4}\cdot\frac{3}{3}\right)$$
$$=-\frac{8}{12}+\left(-\frac{9}{12}\right)$$
$$=-\frac{17}{12}$$

Simplifying a Series of Additions and Subtractions
1. Change all subtractions to additions of opposites.
2. Group and add all of the positive numbers.
3. Group and add all of the negative numbers.
4. Add the results of steps 2 and 3.

Example: Simplify. $6-(-3)-9-(-5)-17$

$$6-(-3)-9-(-5)-17=6+3+(-9)+5+(-17)$$
$$=[6+3+5]+[(-9)+(-17)]$$
$$=14+(-26)$$
$$=-12$$

We know that the terms of an algebraic expression are separated by addition signs. What are the terms of $3x-4y-7$? Start by changing all subtractions to additions of opposites.

$$3x-4y-7=3x+(-4y)+(-7)$$

The terms are $3x$, $-4y$, and -7.

Example: Simplify. $-3k-7n-5k+11n$

$$\begin{aligned}-3k-7n-5k+11n &= -3k+(-7n)+(-5k)+11n\\ &= \left[-3k+(-5k)\right]+\left[(-7n)+11n\right]\\ &= -8k+4n\end{aligned}$$

Section 1.7 Multiplication and Division of Real Numbers

Multiplication is repeated addition. For example, $3(-6)$ means that -6 is added three times.

$$\begin{aligned}3(-6) &= (-6)+(-6)+(-6)\\ &= -18\end{aligned}$$

> **The Product of Two Real Numbers**
> The product of two real numbers with **different signs** is found by multiplying their absolute values. The product is **negative.**
> The product of two real numbers with the **same sign** is found by multiplying their absolute values. The product is **positive.**
> The product of 0 and any real number is 0. Thus, for any real number a, $a\cdot 0=0$ and $0\cdot a=0$

Example: Multiply. **a.** $-\frac{1}{3}\cdot\frac{2}{7}$ **b.** $(-1.2)(-3)$ **c.** $(-487)(0)$

a. $-\frac{1}{3}\cdot\frac{2}{7}=-\frac{2}{21}$

b. $(-1.2)(-3)=3.6$

c. $(-487)(0)=0$

Example: Multiply. $-3(-2)(-4)$

$$-3(-2)(-4) = 6(-4)$$
$$= -24$$

Multiplying More Than Two Numbers

1. Assuming that no factor is zero, the product of an **even** number of **negative numbers** is **positive** and the product of an **odd** number of **negative numbers is negative**. The multiplication is performed by multiplying the absolute values of the given numbers.
2. If any factor is 0, the product is 0.

Example: Multiply. $(-2)(-1)(2)(-3)$

$$(-2)(-1)(2)(-3) = -(2\cdot 6)$$
$$= -12$$

Example: Multiply. $(-2)(-2)(3)(-1)(-3)$

$$(-2)(-2)(3)(-1)(-3) = 4\cdot 9$$
$$= 36$$

Example: Multiply. $(-2)(-5)(0)(-7)(-3)(11)(-6)$

$$(-2)(-5)(0)(-7)(-3)(11)(-6) = 0$$

The result of dividing the real number a by a nonzero real number b is called the **quotient** of a and b. We can write this quotient as $a \div b$ or $\frac{a}{b}$.

We know that subtraction is defined in terms of addition of an additive inverse or opposite: $a - b = a + (-b)$

In a similar way, we can define division in terms of multiplication (you may remember this when working with fractions). For example, $12 \div 3 = 12 \cdot \frac{1}{3}$. We call $\frac{1}{3}$ the *multiplicative inverse*, or *reciprocal*, of 3. Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals** of each other.

Example: Find the multiplicative inverse of each number:

a. 7 **b.** $\frac{1}{5}$ **c.** -8 **d.** $-\frac{3}{4}$

Number	multiplicative inverse
a. 7	$\frac{1}{7}$
b. $\frac{1}{5}$	5
c. -8	$-\frac{1}{8}$
d. $-\frac{3}{4}$	$-\frac{4}{3}$

The number 0 has no multiplicative inverse because 0 multiplied by any number is never 1, but always 0.

> **Definition of Division**
>
> If a and b are real numbers and b is not 0, then the quotient of a and b is defined as $a \div b = a \cdot \frac{1}{b}$
>
> In words: The quotient of two real numbers is the product of the first number and the multiplicative inverse of the second number.

Example: $-26 \div 13$

$$-26 \div 13 = -26 \cdot \frac{1}{13}$$

$$= -\frac{26}{1} \cdot \frac{1}{13}$$

$$= -\frac{26}{13}$$

$$= -2$$

The Quotient of Two Real Numbers

The quotient of two real numbers with **different signs** is found by dividing their absolute values. The quotient is **negative.**

The quotient of two real numbers with the **same sign** is found by dividing their absolute values. The quotient is **positive.**

Division by zero is undefined.

Any nonzero number divided into 0 is 0.

Example: $\frac{9}{-3} = -3$

Example: $-\frac{2}{3} \div \left(-\frac{4}{7}\right) = -\frac{2}{3} \cdot \left(-\frac{7}{4}\right)$

$= \frac{14}{12}$

$= \frac{2 \cdot 7}{2 \cdot 6}$

$= \frac{7}{6}$

Example: $\frac{-10.4}{2} = -5.2$

$$\begin{array}{r} 5.2 \\ 2\overline{)10.4} \\ \underline{-10} \\ 4 \\ \underline{-4} \\ 0 \end{array}$$

Example: $\frac{0}{-5} = 0$

because $-5 \cdot 0 = 0$

Example: $\frac{-5}{0}$ is undefined because 0 times anything is 0. We cannot by 0 to get -5.

$\frac{6}{3} = 2$ because $2 \cdot 3 = 6$

Zero under the line is undefined! Why? We know $\frac{18}{3}=6$ because $3\cdot 6=18$, but what happens when we apply this same idea to $\frac{7}{0}$?

$\frac{7}{0}=?$ because $?\cdot 0=7$ This doesn't make sense. Nothing times 0 is 7.

Additional Properties of Multiplication		
Let a be a real number, a variable, or an algebraic expression.		
Property	Meaning	Examples
Identity Property of Multiplication	1 can be deleted from any product. $a\cdot 1=a$ $1\cdot a=a$	$\sqrt{5}\cdot 1=\sqrt{5}$ $1x=x$ $1(2m-7)=2m-7$
Inverse Property of Multiplication	If a is not 0: $a\cdot\frac{1}{a}=1$ $\frac{1}{a}\cdot a=1$ The product of a nonzero number and its multiplicative inverse, or reciprocal, gives 1, the multiplicative identity.	$3\cdot\frac{1}{3}=1$ $5k\cdot\frac{1}{5k}=1$ $\frac{1}{p+11}\cdot(p+11)=1$ if $p\neq -11$
Multiplication Property of -1	Negative 1 times a is the opposite, or additive inverse, of a. $-1\cdot a=-a$ $a\cdot(-1)=-a$	$-1\cdot\sqrt{5}=-\sqrt{5}$ $-1\left(-\frac{2}{3}\right)=\frac{2}{3}$ $-1x=-x$ $-(g+3)=-1\cdot(g+3)$ $=-g-3$
Double Negative Property	The opposite of $-a$ is a. $-(-a)=a$	$-(-7)=7$ $-(-8w)=8w$

If a negative sign precedes parentheses, remove the parentheses and change the sign of every term within the parentheses.

Examples:

$-(3z+7) = -3z-7$

$-(3z-7) = -3z+7$

$-(-3z+7) = 3z-7$

$-(-3z-7) = 3z+7$

Example: Simplify $-3(5w)$

$$-3(5w) = -15w$$

Example: Simplify $7m+m$

$$7m+m = 8m$$

Example: Simplify $4b-5b$

$$\begin{aligned}4b-5b &= 4b+(-5b)\\ &= -1b\\ &= -b\end{aligned}$$

Example: Simplify $-4(3x-5)$

$$\begin{aligned}-4(3x-5) &= -4(3x)-(-4)(5)\\ &= -12x+20\end{aligned}$$

Example: Simplify $3(2q-4)-(5q-7)$

$$3(2q-4)-(5q-7) = 3\cdot 2q - 3\cdot 4 - 5q + 7$$
$$= 6q - 12 - 5q + 7$$
$$= 6q - 5q + 7 - 12$$
$$= 1q + (-5)$$
$$= q - 5$$

Example: Determine whether -4 is a solution of $-7y+18=-10y+6$

$$-7y+18 = -10y+6$$
$$-7(-4)+18 \stackrel{?}{=} -10(-4)+6$$
$$28+18 \stackrel{?}{=} 40+6$$
$$46 = 46$$

So -4 is a solution.

Section 1.8 Exponents and Order of Operations

base b^p power

An exponent applies only to a base. A negative sign is NOT part of a base unless it appears in parentheses.

Example: Evaluate:

a. 3^2 **b.** $(-2)^3$ **c.** $(-3)^2$ **d.** -3^2

a. $3^2 = 9$

b. $(-2)^3 = (-2)(-2)(-2)$
$= -8$

c. $(-3)^2 = (-3)(-3)$
$= 9$

d. $-3^2 = -(3 \cdot 3)$
$= -9$

Definition of a Natural Number Exponent

If b is a real number and n is a natural number,

exponent

base

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{}$$

b appears as a

factor n times

b^n is read "the nth power of b" or "b to the nth power." Thus, the nth power of b is defined as the product of n factors of b. The expression b^n is called an **exponential expression.**

Furthermore, $b^1 = b$.

The distributive property can be used to simplify certain algebraic expressions.

$$\begin{aligned}3d^2 + 5d^2 &= (3+5)d^2 \\ &= 8d^2\end{aligned}$$

We usually think of this as combining like terms and skip the distributive property step.

Examples:

$5n^3 + 8n^3 =$ $13n^3$

$7m^2 + m^2 =$ $8m^2$

$6r^3 + 5r^2$ cannot be combined

Suppose you want to find the value of $7 + 3 \cdot 9$. What operation do you perform first?

multiplocation

Recall the **order of operations** we introduced at the beginning of the chapter. One of these rules stated that if a problem contains no parentheses or other grouping symbols, perform multiplication before addition. Some problems contain grouping symbols, such as parentheses, brackets, absolute value symbols, fractions bars, or radical symbols. These grouping symbols tell us what to do first.

Order of Operations
1. Perform all operations within grouping symbols.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in the order in which they occur, working from left to right.
4. Do all additions and subtractions using one of the following procedures:
    a. Work from left to right and do the additions and subtractions in the order in which they occur.

    or

    b. Rewrite subtractions as additions of opposites. Combine positive and negative numbers separately, and then add these results.

Pneumonic device: **Please Excuse My Dear Aunt Sally** PEMDAS
Please – **P**arentheses (and other grouping symbols)
Excuse – **E**xponents
My Dear – **M**ultiplication/**D**ivision (work in order from left to right)
Aunt Sally – **A**ddition/**S**ubtraction (work in order from left to right)

Try these:

$36-12\div4+2 = 36-3+2$

$= 33+2$

$= 35$

$10^2-100\div5^2\cdot2-1 = 100-100\div25\cdot2-1$

$= 100-4\cdot2-1$

$= 100-8-1$

$= 92-1$

$= 91$

$(2\cdot3)^2-2\cdot3^2 = 6^2-2\cdot3^2$

$= 36-2\cdot9$

$= 36-18$

$= 18$

$$\left[11-4\left(2-3^3\right)\right]\div 37 = \left[11-4(2-27)\right]\div 37$$

$$= \left[11-4(2+(-27))\right]\div 37$$

$$= \left[11-4(-25)\right]\div 37$$

$$= \left[11+100\right]\div 37$$

$$= 111\div 37$$

$$= 3$$

$$\begin{array}{r} {}^{2}37 \\ \times 3 \\ \hline 111 \end{array}$$

$$\frac{-5(7-2)-3(4-7)}{-13-(-5)} = \frac{-5(5)-3(-3)}{-13+5}$$

$$= \frac{-25+9}{-8}$$

$$= \frac{-16}{-8}$$

$$= 2$$

$$30 \div \frac{5^2}{7-12} - (-9) = 30 \div \frac{25}{-5} - (-9)$$

$$= 30 \div (-5) + 9$$

$$= -6 + 9$$

$$= 3$$

$$\frac{\frac{3}{5} - \frac{7}{10}}{\frac{1}{2}} = \frac{\frac{3}{5} \cdot \frac{2}{2} - \frac{7}{10}}{\frac{1}{2}}$$

$$= \frac{\frac{6}{10} - \frac{7}{10}}{\frac{1}{2}}$$

$$= \frac{-\frac{1}{10}}{\frac{1}{2}}$$

$$= -\frac{1}{10} \cdot \frac{2}{1}$$

$$= -\frac{2}{10}$$

$$= -\frac{2 \cdot 1}{2 \cdot 5}$$

$$= -\frac{1}{5}$$

Some notes:

Note that the equal sign is inline with the main fraction bar.

The main fraction bar should be the longest fraction bar in the problem. Let's look at an example to show that it makes a difference.

$$\frac{\frac{8}{4}}{2} = \frac{2}{2} \quad \text{or} \quad \frac{8}{\frac{4}{2}} = \frac{8}{2}$$

$$= 1 \qquad\qquad = 4$$

Simplify each algebraic expression by removing parentheses and brackets.

$$4[6(x-3)+1] = 4[6x-18+1]$$
$$= 4[6x-17]$$
$$= 4 \cdot 6x - 4 \cdot 17$$
$$= 24x - 68$$

$$6-5[8-(2y-4)] = 6-5[8-2y+4]$$
$$= 6-5[12-2y]$$
$$= 6-5 \cdot 12 - 5(-2y)$$
$$= 6-60+10y$$
$$= -54+10y$$
$$= 10y-54$$

Evaluate $\frac{3m-2m^2}{m(m-2)}$ for $m=5$.

$$\frac{3m-2m^2}{m(m-2)} = \frac{3(5)-2(5)^2}{5(5-2)}$$

$$= \frac{3(5)-2(25)}{5(3)}$$

$$= \frac{15-50}{15}$$

$$= \frac{-35}{15}$$

$$= -\frac{\cancel{5}\cdot 7}{\cancel{5}\cdot 3}$$

$$= -\frac{7}{3}$$

Express the sentence as a single numerical expression and then use the order of operations to simplify the expression.

Subtract 11 from 9. Multiply this difference by 2. Raise this product to the fourth power.

$$[2(9-11)]^4 = [2(-2)]^4$$

$$= (-4)^4$$

$$= 256$$

In Palo Alto, California, a government agency ordered computer-related companies to contribute to a pool of money to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers.) The mathematical model $C = \frac{200x}{100 - x}$ describes the cost, C, in tens of thousands of dollars, for removing x percent of the contaminants.

10 1000

Find the cost, in tens of thousands of dollars, for removing 50% of the contaminants. $x = 50$

$$C = \frac{200(50)}{100-50} = \frac{10,000}{50} = \frac{1000}{5} = 200$$

$(200 \cdot 10,000)$

It would costs $2,000,000 to remove 50% of the contaminants

Find the cost, in tens of thousands of dollars, for removing 80% of the contaminants. $x = 80$

$$C = \frac{200(80)}{100-80} = \frac{16000}{20} = \frac{1600}{2} = 800$$

It would cost $8,000,000 to remove 80% of the contaminants.

Describe what is happening to the cost of the cleanup as the percentage of contaminant removed increases.

As the percent of contaminant removed increases, the cost is growing at an alarming rate.