

Section 1.3 The Real Numbers

A set is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of natural numbers used for counting can be represented by $\{1, 2, 3, 4, 5, \dots\}$. If we add the number 0 to the set of natural numbers, we get the set of **whole numbers** $\{0, 1, 2, 3, 4, 5, \dots\}$. The **integers** include the whole numbers as well as the opposites (negatives) of the natural numbers. The set of integers can be represented as $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$.

The set of **rational numbers** is the set of all numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0, written $b \neq 0$. The integer a is called the **numerator** and the integer b is called the **denominator**.

Example: Express each rational number as a decimal.

a. $\frac{3}{8}$

$$\begin{array}{r} 0.375 \\ 8 \overline{) 3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\frac{3}{8} = 0.375$$

b. $\frac{3}{11}$

$$\begin{array}{r} 0.2727\dots \\ 11 \overline{) 3.0000} \\ \underline{-22} \\ 80 \\ \underline{-77} \\ 30 \\ \underline{-22} \\ 80 \\ \underline{-77} \\ 3 \end{array}$$

$$\frac{3}{11} = 0.\overline{27}$$

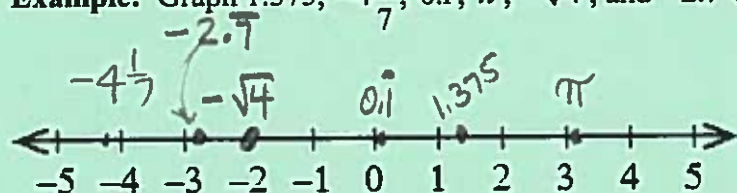
Any rational number can be expressed as a decimal. The resulting decimal will either terminate (stop), or it will have a digit that repeats or a block of digits that repeat.

Any number that can be represented on the number line that is not a rational number is called an **irrational number**. Thus, the set of irrational numbers is the set of numbers whose decimal representations are neither terminating nor repeating.

The **real numbers** are made up of the rational numbers and the irrational numbers.

We generally label the real number line only with integers.

Example: Graph 1.375 , $-4\frac{1}{7}$, $0.\bar{1}$, π , $-\sqrt{4}$, and $-2.\bar{7}$ on a number line.



$$\pi \approx 3.14$$

$$-\sqrt{4} = -2$$

Example: Consider the following set of numbers: $\left\{-4\frac{1}{7}, -2.\bar{3}, -1, 0, \sqrt{2}, \sqrt{4}, \pi, \frac{9}{2}\right\}$

List the numbers in the set that are

a. natural numbers $\sqrt{4}$

b. whole numbers $0, \sqrt{4}$

c. integers $-1, 0, \sqrt{4}$

d. rational numbers $-4\frac{1}{7}, -2.\bar{3}, -1, 0, \sqrt{4}, \frac{9}{2}$

e. irrational numbers $\sqrt{2}, \pi$

f. real numbers the whole set

Example: Write $<$, $>$, or $=$ in between the numbers to make a true statement.

$$0.\bar{45} > \frac{7}{16}$$

$$\begin{array}{r} 0.43 \\ 16 \overline{) 7.00} \\ \underline{-64} \\ 60 \\ \underline{-48} \\ 12 \end{array}$$

$$\begin{array}{r} 2 \\ 16 \\ \times 4 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 16 \\ \times 3 \\ \hline 48 \end{array}$$

$$\pi \approx 3.14$$

Example: Write $<$, $>$, or $=$ in between the numbers to make a true statement.

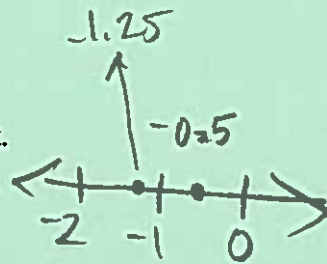
a. $3.14 < \pi$ b. $-4 < 3$ c. $-1.25 < -0.5$

Example: Determine whether each inequality is true or false.

a. $-3 \leq 1$ True

b. $-5 \leq -5$ True

c. $-2 \geq 1$ False



The **absolute value** of a real number a , denoted $|a|$, is the distance from 0 to a on a number line.

Because absolute value describes distance, it is never negative.

Example: $|-7| = 7$

Example: $|\sqrt{3}| = \sqrt{3}$

Example: $|0| = 0$

Example: Write $<$, $>$, or $=$ in between the numbers to make a true statement.

a. $|-5| > |2|$ b. $|-4| < |7|$ c. $|\frac{1}{3}| = |0.\bar{3}|$

$$\frac{1}{3} = 0.\bar{3}$$

Section 1.4 Basic Rules of Algebra

Recall that an algebraic expression combines variables and numbers. Here is an example of an algebraic expression: $2x + 5$

The **terms** of an algebraic expression are those parts that are separated by addition.

The terms $2x + 5$ are $2x$ and 5 .

The numerical part of a term is called a **coefficient**. The coefficient of $2x$ is 2 .

If a term containing one or more variables is written without a coefficient, the coefficient is understood to be 1 . The coefficient of m is 1 . The coefficient of xyz is 1 .

A term that consists of just a number is called a **constant term**. In the algebraic expression $2x + 5$, 5 is a constant term.

The parts of each term that are multiplied are called the **factors of the term**. The factors of $2x$ are 2 and x .

Like terms are terms that have exactly the same variable factors. $2x$ and $5x$ are like terms. $3a^2$ and $4a^2$ are like terms. $7mn$ and mn are like terms. $2xy^2$ and $3xy^2$ are like terms. $2xy^2$ and $3x^2y$ are NOT like terms. Constant terms like 5 and -2 are like terms.

Example: Consider the algebraic expression $4x + 3 + 5x$.

a. How many terms are in the algebraic expression? 3

b. What is the coefficient of the first term? 4

c. What is the constant term? 3

[If we simplified, there would only be 2 terms.]

$$\begin{array}{r} 0.33 \\ 3 \overline{) 1.00} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

d. What are the like terms in the algebraic expression? $4x$ and $5x$

Example: Evaluate each of the algebraic expressions for $x = 2$.

a. $4x + 3 + 5x$

b. $9x + 3$

$$\begin{aligned} \text{a) } 4x + 3 + 5x &= 4(2) + 3 + 5(2) \\ &= 8 + 3 + 10 \\ &= 21 \end{aligned}$$

$$\begin{aligned} \text{b) } 9x + 3 &= 9 \cdot 2 + 3 \\ &= 18 + 3 \\ &= 21 \end{aligned}$$

Regardless of what number you select for x , the algebraic expressions $4x + 3 + 5x$ and $9x + 3$ will have the same value. Two algebraic expressions that have the same value for all replacements are called **equivalent algebraic expressions**.

Properties of Real Numbers and Algebraic Expressions

The Commutative Properties

Let a and b represent real numbers, variables, or algebraic expressions.

Commutative Property of Addition

$$a + b = b + a$$

Changing order when adding does not affect the sum.

Commutative Property of Multiplication

$$ab = ba$$

Changing order when multiplying does not affect the product.

Example: Does the commutative property hold for subtraction? Explain.

No

$$5 - 3 = 2$$

$$3 - 5 = -2$$

Changing the order in which we subtract changes the sign of the result.

Example: Does the commutative property hold for division? Explain.

No

$$\frac{8}{4} = 2$$

$$\frac{4}{8} = \frac{1}{2}$$

We can see from the example that the commutative property does not hold for division.

The Associative Properties

Let a , b , and c represent real numbers, variables, or algebraic expressions.

Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

Changing grouping when adding does not affect the sum.

Associative Property of Multiplication

$$(ab)c = a(bc)$$

Changing grouping when multiplying does not affect the product.

Example: Compare $(2 \cdot 3)4$ and $2(3 \cdot 4)$.

$$\begin{aligned}(2 \cdot 3)4 &= 6(4) \\ &= 24\end{aligned}$$

$$\begin{aligned}2(3 \cdot 4) &= 2(12) \\ &= 24\end{aligned}$$

Example: Does the associative property hold for subtraction? Explain.

No Consider $(5-4)-3$ and $5-(4-3)$

$$\begin{array}{l} (5-4)-3 = 1-3 \\ \quad = -2 \end{array} \qquad \begin{array}{l} 5-(4-3) = 5-1 \\ \quad = 4 \end{array}$$

Example: Does the associative property hold for division? Explain.

No $(8 \div 4) \div 2 = 2 \div 2 = 1$ and $8 \div (4 \div 2) = 8 \div 2 = 4$

The example shows the associative property does not hold for division.

Example: Simplify $5+(3+x)$.

$$\begin{aligned} 5+(3+x) &= (5+3)+x \\ &= 8+x \end{aligned}$$

Example: Simplify $5(3x)$.

$$\begin{aligned} 5(3x) &= (5 \cdot 3)x \\ &= 15x \end{aligned}$$

Example: Simplify $2+(m+9)$.

$$\begin{aligned} 2+(m+9) &= m+(2+9) \\ &= m+11 \end{aligned}$$

The **distributive property** involves both multiplication and addition.

$$\begin{aligned} 3(2+5) &= 3(7) \\ &= 21 \end{aligned}$$

$$\begin{aligned} 3(2+5) &= 3 \cdot 2 + 3 \cdot 5 \\ &= 6 + 15 \\ &= 21 \end{aligned}$$

The Distributive Property

Let a , b and c represent real numbers, variables, or algebraic expressions.

$$a(b+c) = ab+ac$$

Multiplication distributes over addition.

Example: Multiply $3(x+2)$.

$$\begin{aligned} 3(x+2) &= 3 \cdot x + 3 \cdot 2 \\ &= 3x + 6 \end{aligned}$$

Example: Multiply $5(2y-4)$.

$$\begin{aligned} 5(2y-4) &= 5 \cdot 2y - 5 \cdot 4 \\ &= 10y - 20 \end{aligned}$$

The distributive property allows us to **combine like terms**.

$$6x + 2x = (6 + 2)x \\ = 8x$$

When combining like terms, we usually leave out the details of the distributive property.

$$3 \text{ bananas} + 5 \text{ bananas} = 8 \text{ bananas}$$

$$3b + 5b = 8b$$

$$10 \text{ meters} - 4 \text{ meters} = 6 \text{ meters}$$

$$10m - 4m = 6m$$

2 bananas + 7 meters cannot be simplified

$$2b + 7m \text{ can't be simplified}$$

Simplifying Algebraic Expressions

1. Use the distributive property to remove parentheses.
2. Rearrange terms and group like terms using the commutative and associative properties. This step may be done mentally.
3. Combine like terms by combining the coefficients of the terms and keeping the same variable factor.

Example: Simplify $3(6p + 5) + 2p$.

$$\begin{aligned} 3(6p + 5) + 2p &= 3 \cdot 6p + 3 \cdot 5 + 2p \\ &= 18p + 15 + 2p \\ &= 20p + 15 \end{aligned}$$

Example: Simplify $4(2a+6b)+7(4a-3b)$.

$$\begin{aligned}4(2a+6b)+7(4a-3b) &= 4 \cdot 2a + 4 \cdot 6b + 7 \cdot 4a - 7 \cdot 3b \\ &= 8a + 24b + 28a - 21b \\ &= 36a + 3b\end{aligned}$$

Example: Simplify $\frac{1}{3}(12+9k)$.

$$\begin{aligned}\frac{1}{3}(12+9k) &= \frac{1}{3} \cdot \frac{12}{1} + \frac{1}{3} \cdot \frac{9k}{1} \\ &= \frac{12}{3} + \frac{9k}{3} \\ &= 4 + 3k\end{aligned}$$

Example: Simplify $5r+9+7r-3$.

$$\begin{aligned}5r+9+7r-3 &= 5r+7r+9-3 \\ &= 12r+6\end{aligned}$$



Example: Write nine increased by the product of 3 and 2 less than a number as an algebraic expression, then simplify the expression. Don't forget to start by defining a variable.

Let n represent the number.

$$\begin{aligned}9 + 3(n-2) &= 9 + 3 \cdot n - 3 \cdot 2 \\ &= 9 + 3n - 6 \\ &= 3n + 3\end{aligned}$$