Week 4 and week 5 lecture notes

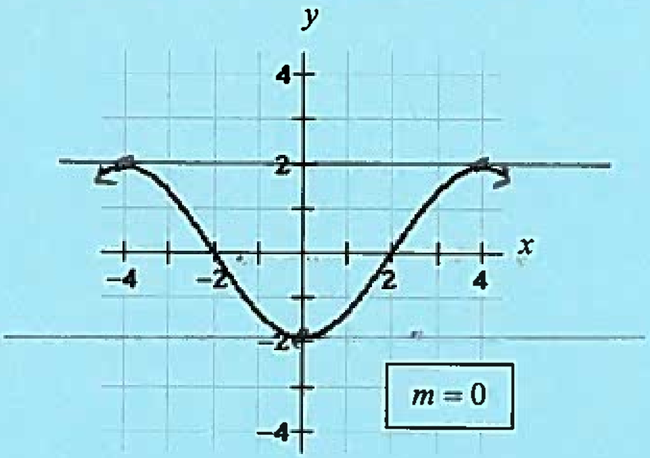


Figure 1: Tan lines to $y = f(x)$

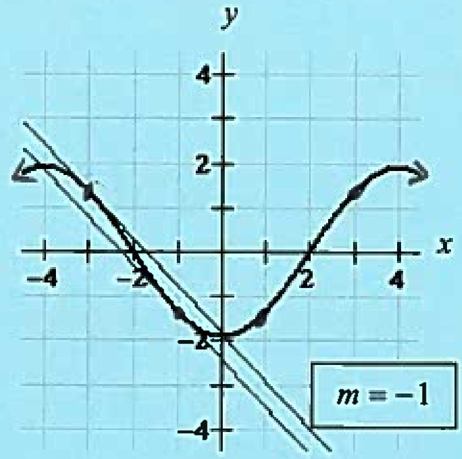


Figure 2: Tan lines to $y = f(x)$

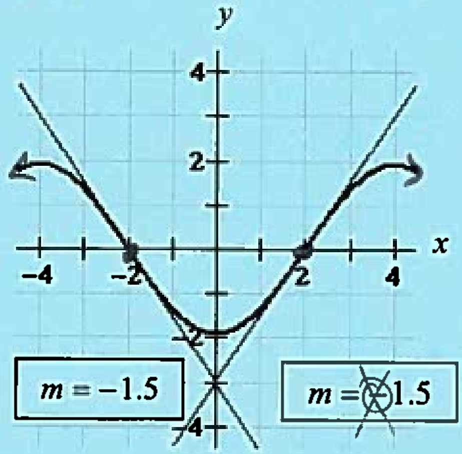


Figure 3: Tan lines to $y = f(x)$

Table 1: Tangent line slopes for $y = f(x)$

x	Figure #	$f'(x)$
−4	1	0
−3	2	−1
−2	3	−1.5
−1	2	−1
0	1	0
1	use symmetry	1
2	3	1.5
3	use symmetry	1
4	1	0

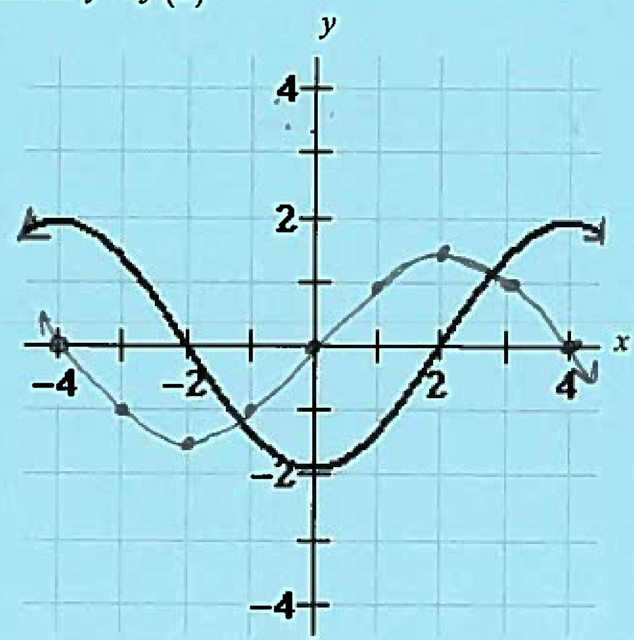


Figure 4

Computer drawn $y = f(x)$

Hand drawn $y = f'(x)$

Let's answer each of the following questions about g over the interval $(-6,6)$.

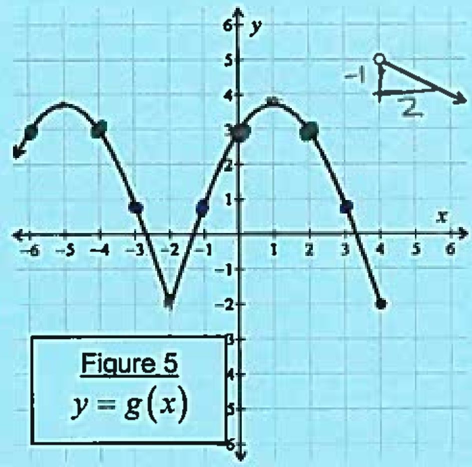


Figure 5
$y = g(x)$

At what values of x does $g'(x) = 0$?

-5 and 1

At what values of x is g nondifferentiable?

-2 and 4

Along what intervals is the value of $g'(x)$ always positive?

$(-6,-5), (-2,1)$

Along what intervals is the value of $g'(x)$ always negative?

$(-5,-2), (1,4), (4,6)$

Along what intervals is the value of $g'(x)$ constant?

$(4,6)$

Three values of $g'(x)$ are given in Table 2. Let's go ahead and complete the rest of the table and then draw the function g' onto Figure 6.

Table 2: $y = g'(x)$

x	-6	-5	-4	-3	-2.01	-1.99	-1	0	1	2	3	4	5
y	1.5	0	-1.5	-2.6	-3	3	2.6	1.5	0	-1.5	-2.6	undefined	$-\frac{1}{2}$

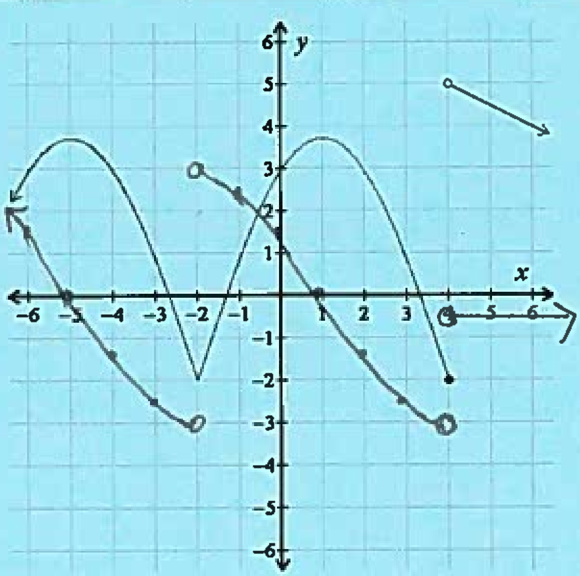


Figure 6
Computer drawn $y = g(x)$
Hand drawn $y = g'(x)$

$$g'(-2) = \lim_{h \to 0} \frac{g(-2+h) - g(-2)}{h}$$

$\lim_{h \to 0^-} \frac{g(-2+h) - g(-2)}{h}$ is close to -3

$\lim_{h \to 0^+} \frac{g(-2+h) - g(-2)}{h}$ is close to 3

$\Bigg\}$ $g'(-2)$ is undefined

A function, f, is nondifferentiable at a if

1. f has a discontinuity at a
2. f has a cusp (sharp corner) at a
3. f has a vertical tangent at a

A function, f, is nondifferentiable at a if $f'(a)$ is undefined.

Decide whether each statement is true or false. If the statement is false, give at least 2 examples of points or intervals from the functions f, g, h, and m shown in figures 6-9 that "prove" the falsity of the statement.

If every tangent line to a function is an increasing line, then the function is always increasing.

True

If a function is always increasing, then its first derivative is always positive.

False m is increasing on (-5, 3), but m'(-1) is undefined and m'(-4) = 0.

If a function's first derivative is always negative, then the function is always decreasing.

True

The first derivative always has a value of 0 at a local minimum or local maximum point.

False g has local maximums at -3 and 2, but g(-3) is undefined and g'(2) is undefined.

A function always has a local maximum point or a local minimum point at a point where its first derivative value is 0.

False h is decreasing on (-∞, -1.5), but h'(-4) = 0.

m is increasing on (-5, 3), but h'(-4) = 0

If a function is always decreasing, then its first derivative is never positive.

True

If a function is always decreasing, then its first derivative is always negative.

False h is decreasing on (-∞, -1.5), but h'(-4) = 0.

m is decreasing on (3, ∞), but m'(5) is undefined

A function is always nondifferentiable at any point it is discontinuous.

True

The first derivative of a linear function is linear.

True (horizontal line)

The steeper the tangent line, the greater the first derivative value.

h is steeper at -6 than at -5, but h'(-6) < h'(-5)

m is steeper at 5 than anywhere else, but m'(5) isn't defined.

Supplemental Graphs for MTH 251 Week 4 Lecture Notes

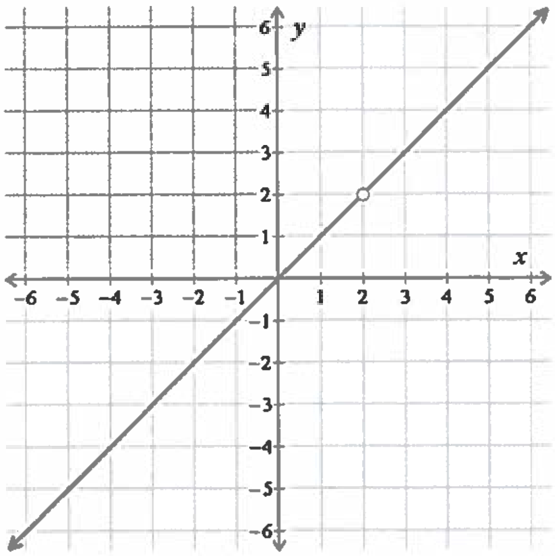


Figure 13: $y = f(x)$

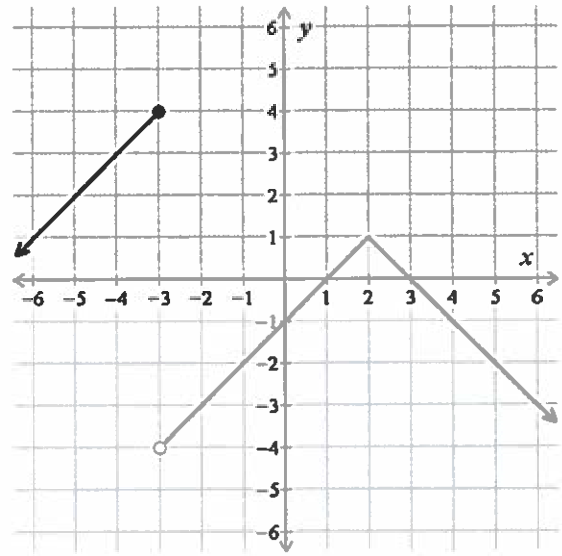


Figure 14: $y = g(x)$

x	$h'(x)$
-5	-3
-4	0
-3	-3
-2	-0.8
-1.5	0
-1	0.8
0	3
1	undefined

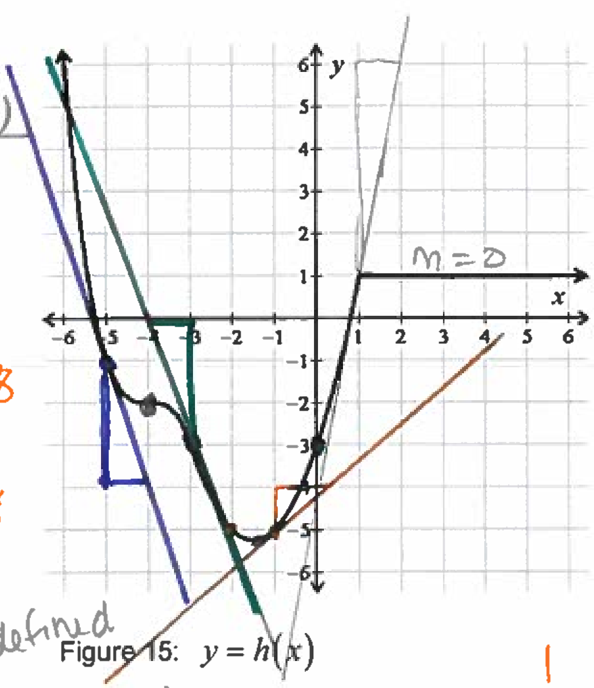


Figure 15: $y = h(x)$

$\frac{1}{1.3}$

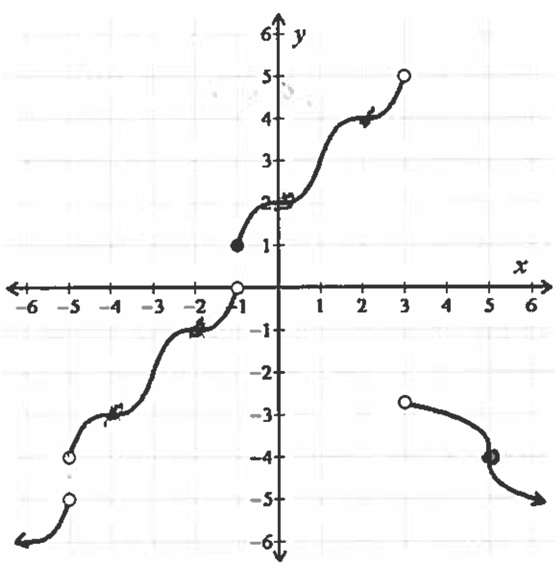
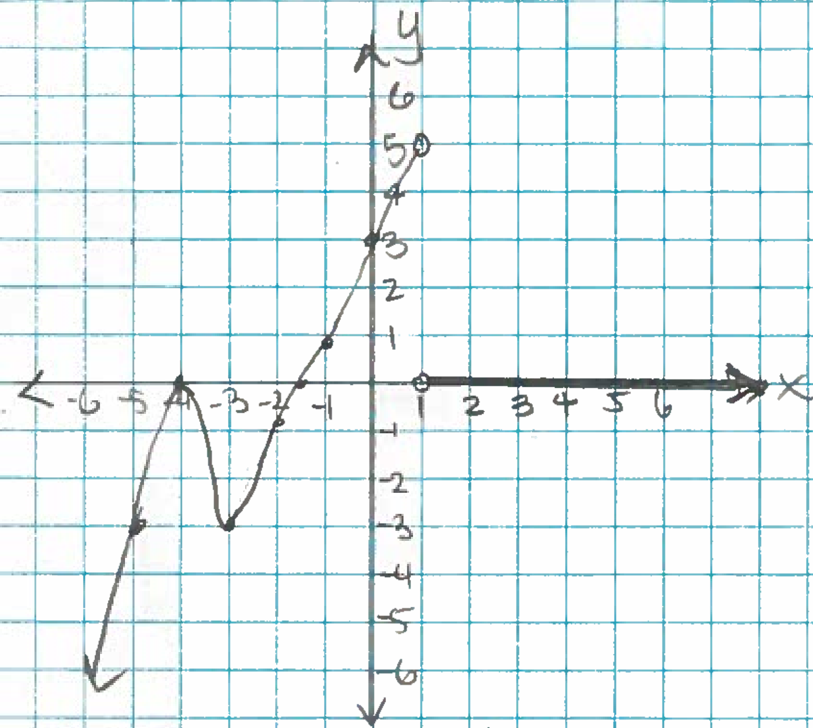


Figure 16: $y = m(x)$

when $x > 1$, $h'(x) = 0$

h' would have holes at $(1,0)$ and $(1,5)$



$y = h'(x)$