Use the graph shown in Figure 1 to estimate the value of $f'(1)$ for the function $f(x) = 4 - x^2$. Next, find the exact value of $f'(1)$ using the formal definition of the first derivative function at a point. Then find the equation of the tangent line to $f(x) = 4 - x^2$ at the point $(1,3)$.

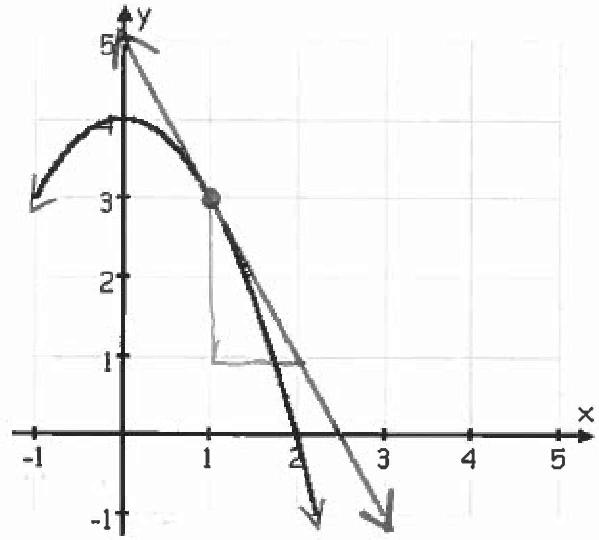


Figure 1: Tangent Line to $y = 4 - x^2$ at the point $(1,3)$

$$f'(1) = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0} \frac{4-(1+h)^2 - (4-1^2)}{h}$$
$$= \lim_{h\to 0} \frac{4-(1+2h+h^2) - 3}{h}$$
$$= \lim_{h\to 0} \frac{1-1-2h-h^2}{h}$$
$$= \lim_{h\to 0} \frac{-2h-h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(-2-h)}{h}$$
$$= \lim_{h\to 0} (-2-h) \quad \text{LLA7}$$
$$= \lim_{h\to 0}(-2) - \lim_{h\to 0} h \quad \text{LLA2}$$
$$= -2 - 0 \quad \text{LL R2 \& R1}$$
$$= -2$$

$$y - y_1 = m(x - x_1)$$
$$m = -2$$
$$(1,3)$$
$$y - 3 = -2(x-1)$$
$$y - 3 = -2x + 2$$
$$y = -2x + 5$$

The equation of the tangent line is $y = -2x + 5$.

← This is an optional step after test 1. We will always show LLA7 and the replacement limit laws

Slope Units

Recalling the classic slope formula "rise over run", it naturally follows that if you are working with an applied function then the units on the *slope* of the function are the "rise units" over the "run units." Since the rise is associated with the output of the function and the run is associated with the input of the function, it follows that the units on the *slope* of an applied function are the function's "output units" over the function's "input units."

Let's find the slope units for each of the following functions; state the practical meaning of the slope.

$y = P(t)$ is the population of Oregon (in million people) t years after 1900. Pretend that the slope *value* is constantly 0.025.

The slope unit is $\frac{\text{million people}}{\text{year}}$.

The slope tells us that the population is increasing at a rate of 0.025 $\frac{\text{million people}}{\text{year}}$.

$\left(25000\ \frac{\text{people}}{\text{year}}\right)$

$v = f(t)$ is the velocity of Newton's apple (in ft/sec) t seconds after the apple began its plunge towards earth. Pretend that the slope *value* is constantly -32.1.

The slope unit is $\frac{\text{ft/sec}}{\text{sec}}$.

The slope tells us that the velocity is decreasing at a rate of 32.1 $\frac{\text{ft/sec}}{\text{sec}}$.

Interpreting slopes "fill in the blank"

The slope tell us that $\underset{\text{the function output}}{\underline{\hspace{3cm}}}$ is in/decreasing at a rate of $\underset{|\text{slope}|}{\underline{\hspace{2cm}}}$ $\underset{\text{slope unit}}{\underline{\hspace{2cm}}}$.

$f'(1)$ The first derivative of f evaluated at 1.
Usually read as "f prime of 1."

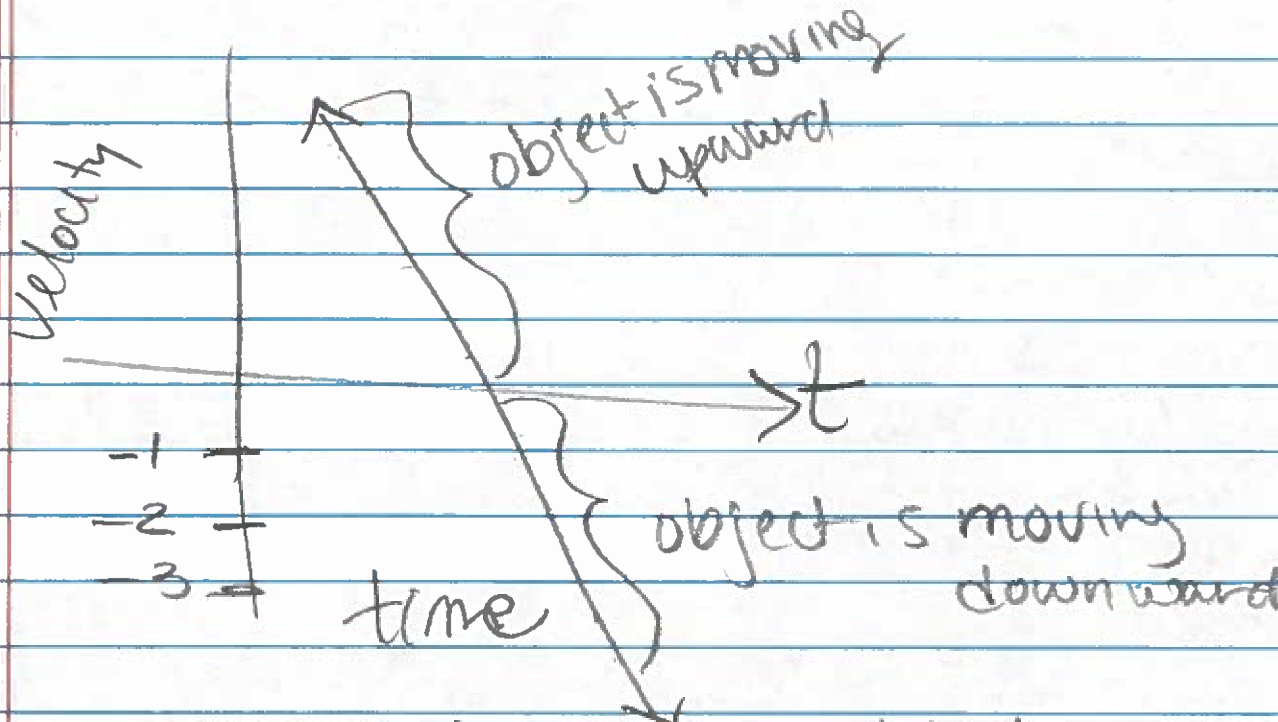
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f'(1)$ is the slope of the tangent line to f at the point where $x=1$.

An alternate method to find the equation of the tangent line:

$y = mx + b$
$m = -2$
$(1, 3)$
$3 = -2(1) + b$
$3 = -2 + b$
$5 = b$

The equation of the tangent line is $y = -2x + 5$.



velocity of an object

The slope of this velocity function is always negative.

On test

tables

Limits: graphically, numerically, and algebraically

Difference quotients (no limit)

continuity

Memorize: 3 conditions for continuity at a point

types: infinite, jump, or removable

drawing graphs based on conditions

technique: multiply by conjugate

technique: limits where the target value is $\pm\infty$, look for highest power (or dominant term) of denominator

multiply by $\dfrac{\frac{1}{\text{highest power}}}{\frac{1}{\text{highest power}}}$

$m(x) = 7$

Find the difference quotient

$$\frac{m(x+h) - m(x)}{h} = \frac{7 - 7}{h}$$

$$= \frac{0}{h}$$

$$= 0, \text{ for } h \neq 0$$

Lab problems I would have collected if I were collecting homework on Tuesday

p. 17	2.3.1	# 7
p. 21	2.6.1	# 1
p. 28 or 29	2.11.1	one of 9, 10, or 11