Decide whether each of the statements on pages 1 and 2 is true or false. For each false statement, state the reason the statement is false.

$$\lim_{t \to 4} \frac{t+6}{t-7} = \frac{\lim_{t \to 4}(t+6)}{\lim_{t \to 4}(t-7)}$$

True by Limit Law A 5

LLA5

$$\lim_{x \to 5} \frac{x^2 + x - 30}{x^2 - 12x + 35} = \frac{\lim_{x \to 5}(x^2 + x - 30)}{\lim_{x \to 5}(x^2 - 12x + 35)}$$

False

LLA5 does not apply because $\lim_{x \to 5}(x^2 - 12x + 35) = 0$.

$$\lim_{x\to\infty}\frac{1}{x^2}=\frac{\lim\limits_{x\to\infty} 1}{\lim\limits_{x\to\infty} x^2}$$

$\lim\limits_{x\to\infty} x^2=\infty$

False
LLA5 does not apply because $\lim\limits_{x\to\infty} x^2$ does not exist.

$\lim\limits_{x\to\infty}\frac{1}{x^2}=0$ by LLR3

$$\lim_{t\to -2}\frac{(3-t)(2+t)}{3-t}=\lim_{t\to -2} 2+t$$

False
The expression on the right needs parenthes around the $2+t$ to make this a true statement.

$\lim\limits_{t\to -2}\frac{(3-t)(2+t)}{3-t}=\lim\limits_{t\to -2}(2+t)$ by LLA7

LLA5 must always be applied in a step by itself.

Symbolically establish the value of $\lim_{x \to 5} \frac{x^2 + x - 30}{x^2 - 12x + 35}$. Show each step in the algebraic simplification. State each limit law used. Organize your work in a manner consistent with the documentation guidelines for MTH 251.

$$\lim_{x \to 5} \frac{x^2+x-30}{x^2-12x+35} = \lim_{x \to 5} \frac{(x+6)(x-5)}{(x-7)(x-5)}$$

$$= \lim_{x \to 5} \frac{x+6}{x-7} \quad \text{by LLA7}$$

$$= \frac{\lim_{x \to 5}(x+6)}{\lim_{x \to 5}(x-7)} \quad \text{LLA5}$$

$$= \frac{\lim_{x \to 5} x + \lim_{x \to 5} 6}{\lim_{x \to 5} x - \lim_{x \to 5} 7} \quad \text{LLA1 \& A2}$$

$$= \frac{5+6}{5-7} \quad \text{LLR1 \& R2}$$

$$= -\frac{11}{2}$$

Multiply by the conjugate to create a difference of squares.

Symbolically establish the value of $\lim_{t\to 49}\frac{\sqrt{t}-7}{49-t}$. Show each step in the algebraic simplification.

State each limit law used. Organize your work in a manner consistent with the documentation guidelines for MTH 251.

$$\lim_{t\to 49}\frac{\sqrt{t}-7}{49-t} = \lim_{t\to 49}\left[\frac{\sqrt{t}-7}{49-t}\cdot\frac{\sqrt{t}+7}{\sqrt{t}+7}\right]$$

$$= \lim_{t\to 49}\frac{t-49}{(49-t)(\sqrt{t}+7)}$$

$$= \lim_{t\to 49}\frac{-1(-t+49)}{(49-t)(\sqrt{t}+7)}$$

$$= \lim_{t\to 49}\frac{-1}{\sqrt{t}+7} \qquad \text{by LLA7}$$

$$= \frac{\lim_{t\to 49}(-1)}{\lim_{t\to 49}(\sqrt{t}+7)} \qquad \text{by LLA5}$$

$$= \frac{-1}{\lim_{t\to 49}\sqrt{t}+\lim_{t\to 49}7} \qquad \text{by LLR2 \& A1}$$

$$= \frac{-1}{\sqrt{\lim_{t\to 49}t}+7} \qquad \text{by LL Ab \& R2}$$

$$= \frac{-1}{\sqrt{49}+7} \qquad \text{by LLR1}$$

$$= -\frac{1}{14}$$

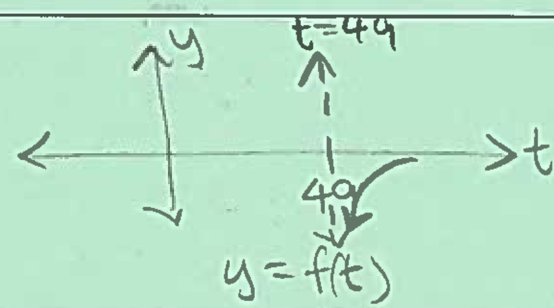
Use a table of values to find $\lim_{t \to 49^+} \frac{\sqrt{t}+7}{49-t}$. Does the limit value exist?

Table 1: $f(t) = \frac{\sqrt{t}+7}{49-t}$

t	$f(t)$
49.1	-140
49.01	-1,400
49.001	-14,000
49.0001	-140,000

Table shows that $\lim_{t \to 49^+} \frac{\sqrt{t}+7}{49-t} = -\infty$ which means this limit doesn't exist.

Side note: $f(t) = \frac{\sqrt{t}+7}{49-t}$ has a vertical asymptote at $t = 49$.



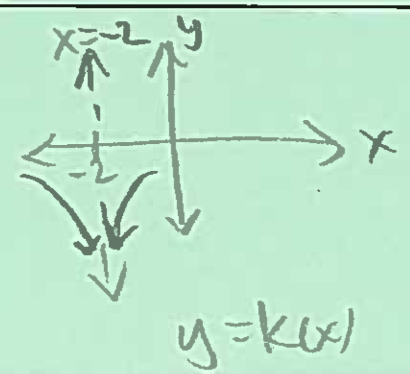
Use a table of values to find $\lim_{x \to -2} \frac{x}{(x+2)^2}$. Does the limit value exist?

Table 2: $K(x) = \frac{x}{(x+2)^2}$

x	$K(x)$
-2.1	-210
-2.01	-20,100
-2.001	-2001,000
-1.999	-1,999,000
-1.99	-19,900
-1.9	-190

Table 2 shows that $\lim_{x \to -2} \frac{x}{(x+2)^2} = -\infty$ which means this limit doesn't exist.

Side note: $K(x) = \frac{x}{(x+2)^2}$ has a vertical asymptote at $x = -2$.



Use a table of values to find $\lim\limits_{u \to 4} \dfrac{5}{u^2 - 6u + 8}$.
Does the limit value exist?

Table 3: $r(u) = \dfrac{5}{u^2-6u+8}$

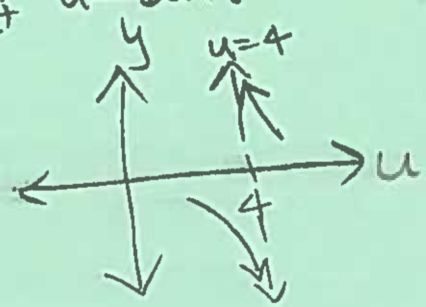
u	$r(u)$
3.99	-251
3.999	-2501
3.9999	-25,001
4.0001	24,999
4.001	2499
4.01	249

Table 3 shows that $\lim\limits_{u \to 4} \dfrac{5}{u^2-6u+8}$ does not exist.

Side note:

$\lim\limits_{u \to 4^-} \dfrac{5}{u^2-6u+8} = -\infty$

$\lim\limits_{u \to 4^+} \dfrac{5}{u^2-6u+8} = \infty$



$y = r(u)$

r has a vertical asymptote at $u=4$.

Use a table of values to find $\lim\limits_{x \to \infty} \dfrac{(x+3)(x-7)}{(4x-6)(x+2)}$.

Table 4: $z(x) = \dfrac{(x+3)(x-7)}{(4x-6)(x+2)}$

x	$z(x)$
1,000	0.24887
10,000	0.249887
100,000	0.2499887
1,000,000	0.24999887

Table 4 shows that

$\lim\limits_{x \to \infty} \dfrac{(x+3)(x-7)}{(4x-6)(x+2)} = 0.25$.

Highest power term in the denominator

Symbolically establish the value of $\lim_{x\to\infty}\frac{(x+3)(x-7)}{(4x-6)(x+2)}$. **Show each step in the algebraic simplification. State each limit law used. Organize your work in a manner consistent with the documentation guidelines for MTH 251.**

$$\lim_{x\to\infty}\frac{(x+3)(x-7)}{(4x-6)(x+2)} = \lim_{x\to\infty}\left[\frac{x^2-4x-21}{4x^2+2x-12}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right]$$

$$= \lim_{x\to\infty}\frac{1-\frac{4}{x}-\frac{21}{x^2}}{4+\frac{2}{x}-\frac{12}{x^2}}$$

$$= \frac{\lim_{x\to\infty}\left(1-\frac{4}{x}-\frac{21}{x^2}\right)}{\lim_{x\to\infty}\left(4+\frac{2}{x}-\frac{12}{x^2}\right)} \qquad \text{LLA5}$$

$$= \frac{\lim_{x\to\infty}1-\lim_{x\to\infty}\frac{4}{x}-\lim_{x\to\infty}\frac{21}{x^2}}{\lim_{x\to\infty}4+\lim_{x\to\infty}\frac{2}{x}-\lim_{x\to\infty}\frac{12}{x^2}} \qquad \text{LLA1 \& A2}$$

$$= \frac{1-0-0}{4+0-0} \qquad \text{LL R2 + R3}$$

$$= \frac{1}{4}$$

* memorize these 3 conditions for test 1!

Definition

The function $f(t)$ is continuous at the number a if and only if each of the following is true.

i. $f(a)$ is defined   ii. $\lim_{t\to a} f(t)$ exists   iii. $\lim_{t\to a} f(t) = f(a)$

The function shown in Figure 1 has several points of discontinuity. For each point of discontinuity state each condition of continuity at a point that fails.

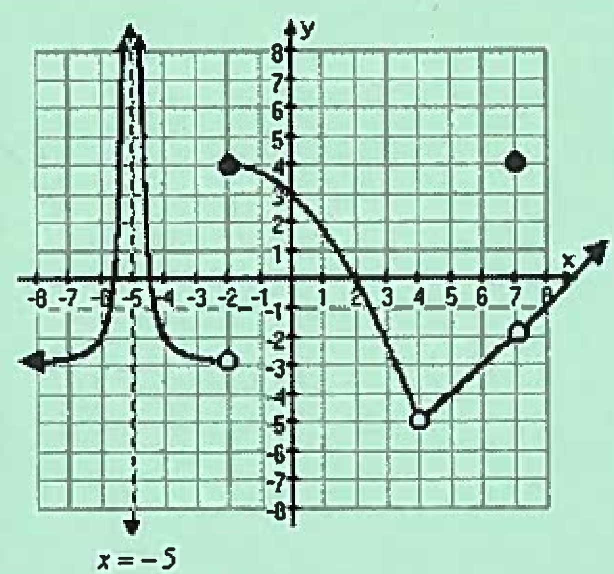


Figure 1: $y = g(x)$

$x = -5$

i) $g(-5)$ is undefined

ii) $\lim_{x\to -5} g(x) = \infty$

iii) $\lim_{x\to -5} g(x) \neq g(-5)$

$g(-2) = 4$ $\lim_{x\to -2^-} g(x) = -3$, but $\lim_{x\to -2^+} g(x) = 4$,

so $\lim_{x\to -2} g(x)$ does not exist.

x-value of discontinuity	Conditions of continuity which fail	Type of discontinuity
−5	i, ii, iii	infinite
−2	ii, iii	jump
4	i, iii	removable
7	iii	removable

$y = \dfrac{x^2-6x+8}{x-2}$ has a removable discontinuity at $x = 2$

$\dfrac{x^2-6x+8}{x-2} = \dfrac{\cancel{(x-2)}(x-4)}{\cancel{x-2}}$ "remove" the common facto

$y(x) = x-4$

Sketch onto Figure 2 one function $y = f(x)$ that satisfies all of the following properties. Do not introduce any unnecessary discontinuities or intercepts that are not directly implied by the stated properties. Make sure that you draw all implied asymptotes and label them with their equations.

- $f(4) = f(1) = f(-5) = 0$

  (4,0)
  (1,0)
  (-5,0)

- $\lim_{x \to 0} f(x) = -\infty$

  f has a vertical asymptote at x=0

- $\lim_{x \to 3^-} f(x) = \infty$ and $\lim_{x \to 3^+} f(x) = -\infty$

  f has a vertical asymptote at x=3

- $\lim_{x \to -\infty} f(x) = -2$

  f has a horizontal asymptote at y=-2

- $\lim_{x \to \infty} f(x) = \infty$

  right hand end behavior

  or or

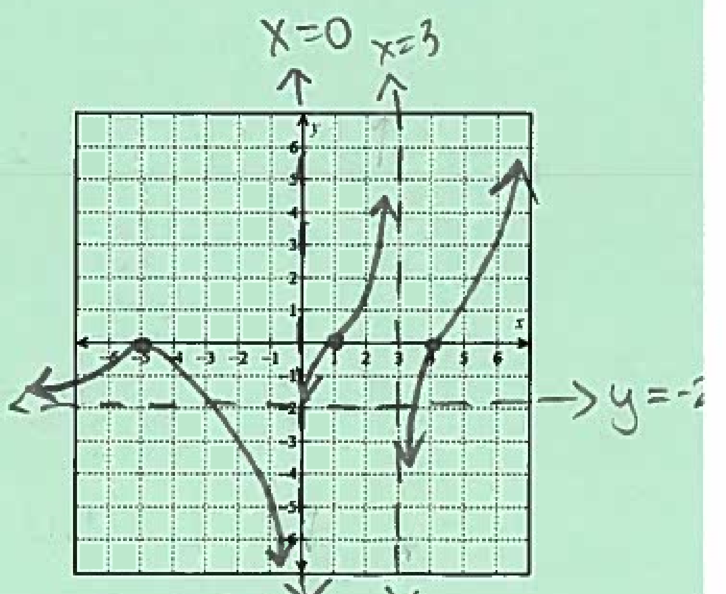


Figure 2: $y = f(x)$

$\Rightarrow$ means "implies that"

$\therefore$ means "therefor"

Find the value of c that makes the function continuous at $x = 4$.

☆ Are there any points at which the function is discontinuous? ☆

$$k(x) = \begin{cases} \dfrac{3}{x-2}; & x < 4 \\ \dfrac{c}{x+7}; & x \geq 4 \end{cases}$$

In order for k to be continuous at x = 4 it must be the case that ① k(4) is defined, ② $\lim_{x \to 4} k(x)$ exists, and ③ $\lim_{x \to 4} k(x) = k(4)$.

If $\lim_{x \to 4} k(x)$ exists, then $\lim_{x \to 4^-} k(x) = \lim_{x \to 4^+} k(x)$

$$\Rightarrow \lim_{x \to 4^-} \frac{3}{x-2} = \lim_{x \to 4^+} \frac{c}{x+7}$$

$$\Rightarrow \frac{3}{4-2} = \frac{c}{4+7}$$

$$\Rightarrow \frac{3}{2} = \frac{c}{11}$$

$$\Rightarrow \frac{33}{2} = c$$

If $c = \frac{33}{2}$, then ① $k(4) = \frac{33/2}{4+7} = \frac{33/2}{11} = \frac{33}{2} \cdot \frac{1}{11} = \frac{3}{2}$

and ② $\lim_{x \to 4^-} k(x) = \lim_{x \to 4^-} \frac{3}{x-2} = \frac{3}{4-2} = \frac{3}{2}$ and $\lim_{x \to 4^+} k(x) = \lim_{x \to 4^+} \frac{33/2}{x+7} = \frac{33/2}{4+7} = \frac{3}{2}$

So $\lim_{x \to 4} k(x) = \frac{3}{2}$.

and ③ $\lim_{x \to 4} k(x) = k(4) = \frac{3}{2}$.

$\therefore$ k is continuous at 4 if $c = \frac{33}{2}$.

continued

☆ Potential discontinuities

$x = 2$ $x = -7$

k has a discontinuity at 2 since k(2) is undefined

$k(2) = \frac{3}{2-2}$

$= \frac{3}{0}$

However k does NOT have a discontinuity at -7 since we use the formula $k(x) = \frac{3}{x-2}$ when $x < 4$ and $-7 < 4$.