The limit as x approaches 2 of $\frac{x^2-6x+8}{x-2}$.

1. Fill Table 1 with values with which you can infer the value of $\lim_{x\to 2}\frac{x^2-6x+8}{x-2}$. State your conclusion using a complete sentence.

Table 1: $y=\frac{x^2-6x+8}{x-2}$

x	y
1.9	-2.1
1.99	-2.01
1.999	-2.001
2.001	-1.999
2.01	-1.99
2.1	-1.9

Table 1 shows that

$$\lim_{x\to 2}\frac{x^2-6x+8}{x-2}=-2.$$

(see page 9)

The limit as x approaches 0 from the left of $\frac{\sin(x)}{x}$

one-sided limit

2. Fill Table 2 with values with which you can infer the value of $\lim_{x\to 0^-}\frac{\sin(x)}{x}$. State your conclusion using a complete sentence.

Table 2: $f(x)=\frac{\sin(x)}{x}$

x	f(x)
-0.1	0.9983
-0.01	0.999983
-0.001	0.9999983

Table 2 infers that

$$\lim_{x\to 0^-}\frac{\sin(x)}{x}=1.$$

3. Fill Table 3 with values with which you can infer the value of $\lim_{t\to 4^+}\left(\ln(t-4)\cdot\sqrt{t-4}\right)$. State your conclusion using a complete sentence.

Table 3: $g(t)=\ln(t-4)\cdot\sqrt{t-4}$

t	g(t)
4.00001	-0.036
4.0000001	-0.005
4.000000001	-0.0007

Table implies that

$$\lim_{t\to 4^+}\left(\ln(t-4)\cdot\sqrt{t-4}\right)=0.$$

4. Figure 1 shows the function $y = g(t)$. Find each of the following values.

$g(-1) = 3$

$\lim_{t \to -1^+} g(t) = 3$

$\lim_{t \to -1^-} g(t) = -3$

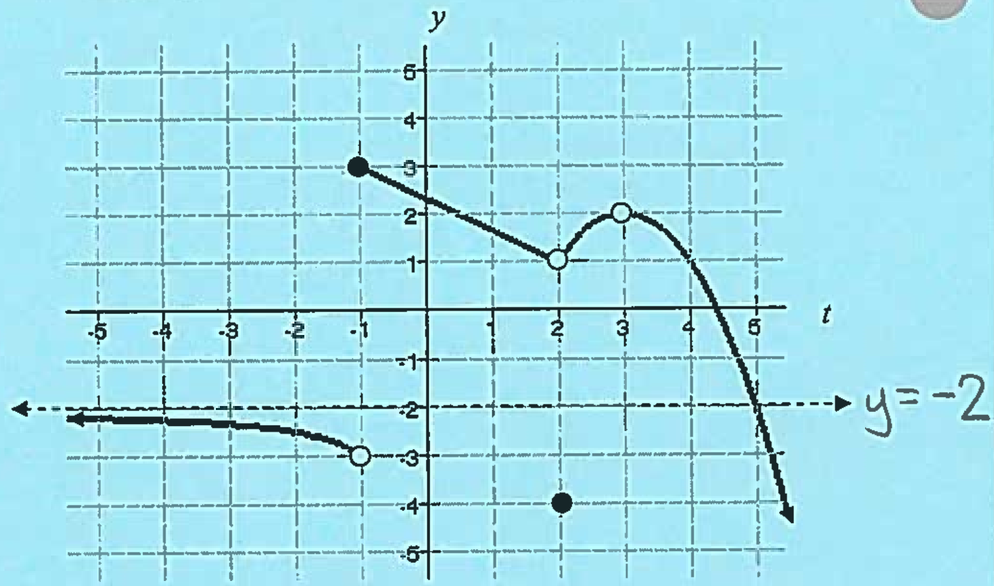


Figure 1: $y = g(t)$

$g(3)$ is undefined

$g(2) = -4$

$\lim_{t \to \infty} g(t) = -\infty$

$\lim_{t \to 3^+} g(t) = 2$

$\lim_{t \to 2^+} g(t) = 1$

$\lim_{t \to -\infty} g(t) = -2$

$\lim_{t \to 3^-} g(t) = 2$

$\lim_{t \to 2^-} g(t) = 1$

For each of the following limits state the value of the limit or state that the limit does not exist as well as the reason the limit does not exist.

$\lim_{t \to 3} g(t)$, $\lim_{t \to -1} g(t)$, $\lim_{t \to -\infty} g(t)$, and $\lim_{t \to \infty} g(t)$

$\lim_{t \to 3} g(t) = 2$

$\lim_{t \to -1} g(t)$ does not exist because $\lim_{t \to -1^+} g(t) \neq \lim_{t \to -1^-} g(t)$.

$\lim_{t \to -\infty} g(t) = -2$

$\lim_{t \to \infty} g(t)$ does not exist because as t increases without bound, g(t) decreases without bound.

as $t \to \infty$, $g(t) \to -\infty$

A **tangent line** to a circle touches the circle at exactly one point.

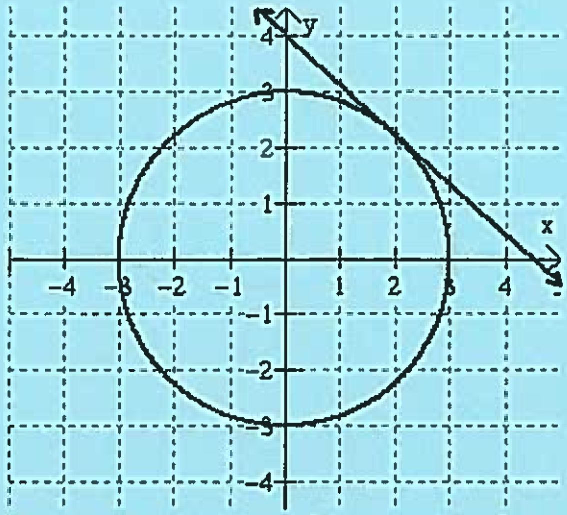


Figure 2: A tangent line to a circle

A **tangent line** to a curve is a little bit harder to define. We will spend the next couple of weeks developing a formal method for finding the slope of a tangent line to a curve at a particular point. We will start with a definition of a **secant line.** A **secant line** to a curve is a line determined by two points on a curve. Figure 3 shows an example of a secant line to a curve through the points $(1,0)$ and $(2,-3)$. We might use this secant line to help us approximate the slope of the tangent line to the curve at the point $(1,0)$.

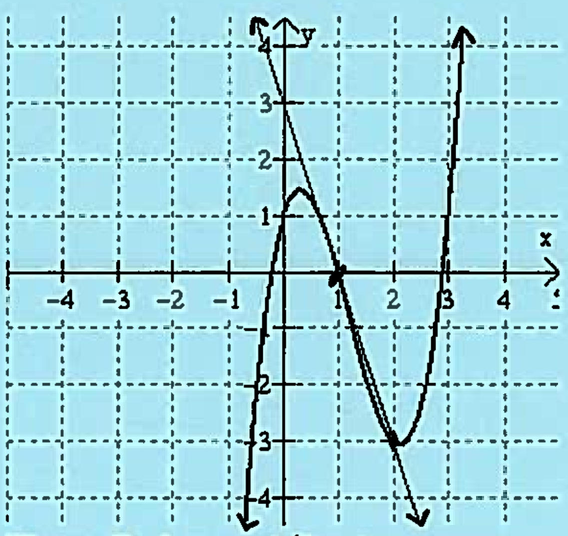


Figure 3: A secant line to a curve

Velocity is a speed with a direction

|velocity| = speed

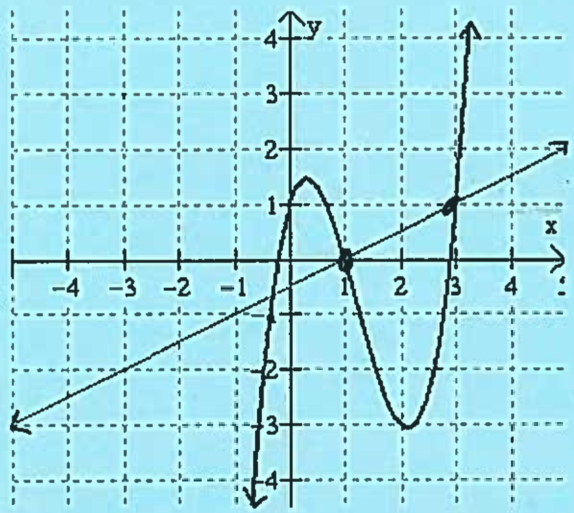


Figure 4: Secant Line through $(1, f(1))$ and $(3, f(3))$

The slope of this secant line is the average velocity from 1 to 3 seconds.

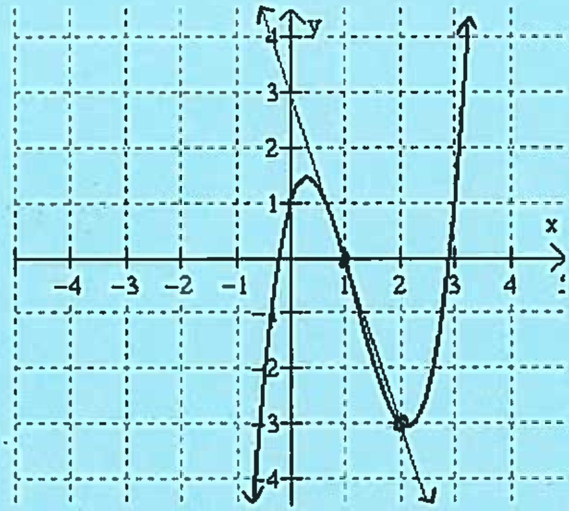


Figure 5: Secant Line through $(1, f(1))$ and $(2, f(2))$

The slope of this secant line is the average velocity from 1 to 2 seconds.

Figures 4 through 7 show the position (in feet) of an object, $y = f(x)$, at time x (in seconds).

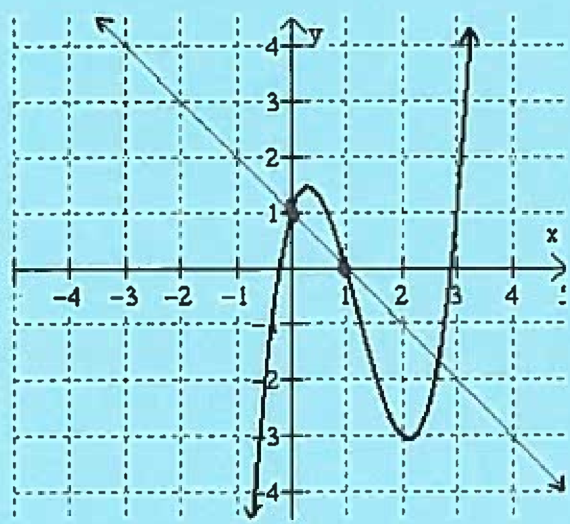


Figure 6: Secant Line through $(1, f(1))$ and $(0, f(0))$

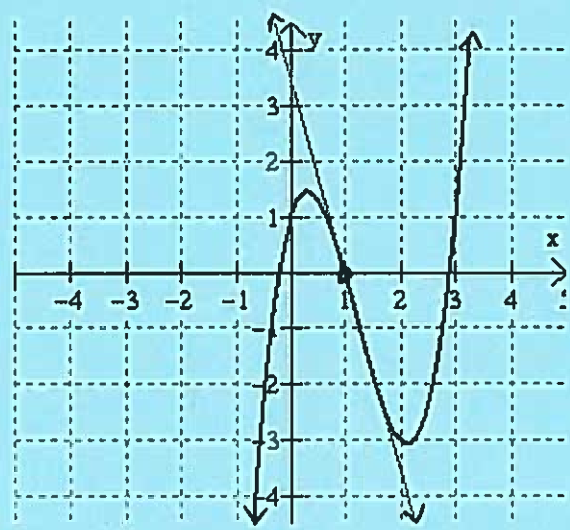


Figure 7: Tangent Line through $(1, f(1))$

The slope of this tangent line is the instantaneous velocity of the object at 1 second.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

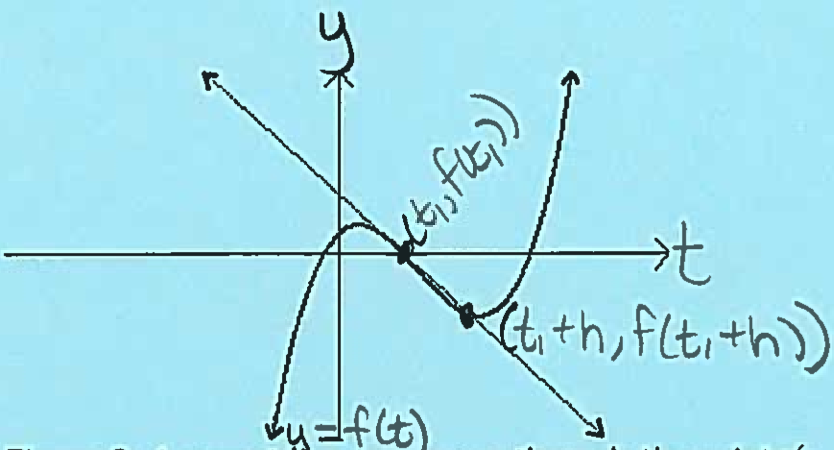


Figure 8: A secant line to a curve through the points $(t_1, f(t_1))$ and $(t_1 + h, f(t_1 + h))$

The slope of the secant line from the point $(t_1, f(t_1))$ to $(t_1 + h, f(t_1 + h))$ is

$$m_{sec} = \frac{\Delta f}{\Delta t}$$

Δ read "delta"
Means "change in"

$$= \frac{f(t_1+h) - f(t_1)}{t_1 + h - t_1}$$

$$= \frac{f(t_1+h) - f(t_1)}{h}$$

This slope is the average rate of change of the function on the interval t_1 to $t_1 + h$.

$$m = \frac{f(t+h) - f(t)}{h}$$

This is called the difference quotient.

Find the difference quotient of each function *paying attention to your presentation.*

a. $f(t) = t^2 + 4t - 7$

$$\frac{f(t+h)-f(t)}{h} = \frac{\left[(t+h)^2+4(t+h)-7\right]-(t^2+4t-7)}{h}$$

$$= \frac{\cancel{t^2}+2th+h^2+\cancel{4t}+4h-\cancel{7}-\cancel{t^2}-\cancel{4t}+\cancel{7}}{h}$$

$$= \frac{2th+h^2+4h}{h}$$

$$= \frac{h(2t+h+4)}{h}$$

$$= 2t+h+4, \text{ for } h \neq 0$$

$$A^2 - B^2 = (A+B)(A-B)$$

b. $g(x) = \dfrac{1}{(x-3)^2}$

$$\frac{g(x+h)-g(x)}{h} = \frac{\frac{1}{(x+h-3)^2} - \frac{1}{(x-3)^2}}{h}$$

$$= \frac{\frac{1}{(x+h-3)^2}\cdot\frac{(x-3)^2}{(x-3)^2} - \frac{1}{(x-3)^2}\cdot\frac{(x+h-3)^2}{(x+h-3)^2}}{h}$$

$$= \frac{\frac{(x-3)^2-(x+h-3)^2}{(x+h-3)^2(x-3)^2}}{h}$$

$$= \frac{\frac{[(x-3)+(x+h-3)][(x-3)-(x+h-3)]}{(x+h-3)^2(x-3)^2}}{h}$$ Factor the difference of squares

$$= \frac{\frac{(x-3+x+h-3)(\cancel{x}\cancel{-3}\cancel{-x}-h\cancel{+3})}{(x+h-3)^2(x-3)^2}}{h}$$

$$= \frac{\frac{(2x+h-6)(-h)}{(x+h-3)^2(x-3)^2}}{h}$$

$$= \frac{-\cancel{h}(2x+h-6)}{(x+h-3)^2(x-3)^2}\cdot\frac{1}{\cancel{h}}$$

$$= \frac{-(2x+h-6)}{(x+h-3)^2(x-3)^2}, \text{ for } h \neq 0$$

(See page 9 & 10)

6. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after t seconds is given by $h = 58t - 0.83t^2$.
   a. Find the average velocity over the given time intervals.
      (i) [1, 2] (ii) [1, 1.5] (iii)[1, 1.1] (iv)[1, 1.01] (v) [1, 1.001]
   b. Estimate the instantaneous velocity one second after the arrow was shot.

$h(t) = 58t - 0.83t^2$

$$V_{\substack{ave \\ [1,2]}} = \frac{\Delta h(t)}{\Delta t} = \frac{h(2)-h(1)}{2-1}$$

Table 6: $h = 58t - 0.83t^2$

time interval (seconds)	average velocity $\frac{m}{s}$
[1, 2]	55.51
[1, 1.5]	55.925
[1, 1.1]	56.257
[1, 1.01]	56.3317
[1, 1.001]	56.33917

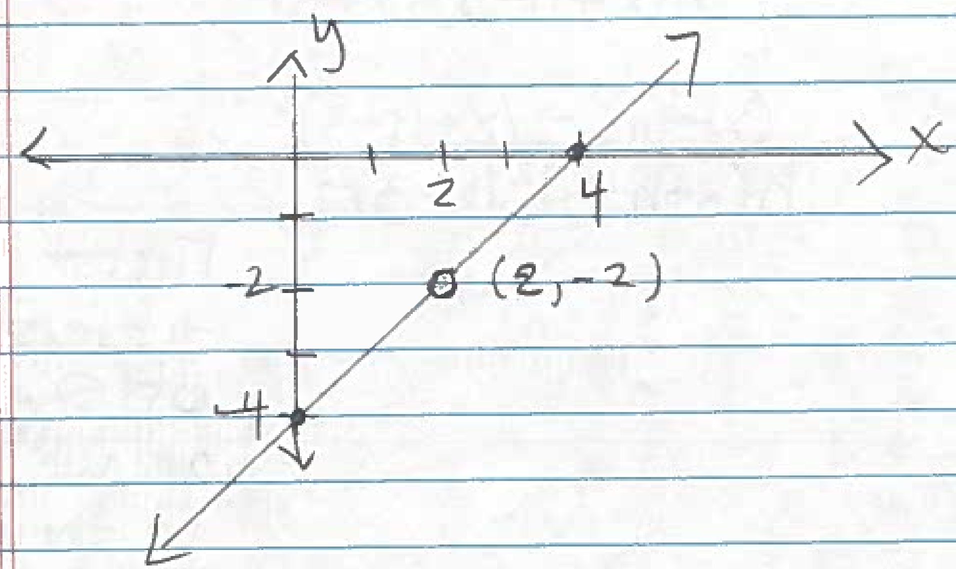
b. Based on Table 6, I estimate the instantaneous velocity one second after the arrow was shot to be about $56.3 \frac{m}{s}$.

page 1 - problem 1 - visual

$$y = \frac{x^2-6x+8}{x-2}$$

$$= \frac{(x-2)(x-4)}{x-2}$$

$$= x-4, \text{ for } x \neq 2$$



page 7 - note

The length of the fraction bar makes a difference!

$$\frac{\frac{8}{4}}{2} \neq \frac{8}{\frac{4}{2}}$$

$$\frac{\frac{8}{4}}{2} = \frac{2}{2} = 1 \quad \text{and} \quad \frac{8}{\frac{4}{2}} = \frac{8}{2} = 4$$

The equal sign should be inline with the main fraction bar when simplifying a complex fraction.

page 7 – Alternate method

$$\frac{g(x+h)-g(x)}{h} = \frac{\frac{1}{(x+h-3)^2} - \frac{1}{(x-3)^2}}{h} \cdot \frac{(x+h-3)^2(x-3)^2}{(x+h-3)^2(x-3)^2}$$

$$= \frac{\frac{1}{\cancel{(x+h-3)^2}} \cdot \cancel{(x+h-3)^2}(x-3)^2 - \frac{1}{(x-3)^2} \cdot (x+h-3)^2(x-3)^2}{h(x+h-3)^2(x-3)^2}$$

$$= \frac{(x-3)^2 - (x+h-3)^2}{h(x+h-3)^2(x-3)^2}$$

$$\vdots$$

Factor the difference of squares as we did in the first method.