Practice tests

EXAMPLE: Find $\lim_{n \to 3^+} \frac{7}{n-3}$ without your calculator.

rational limit form $\frac{7}{0^+}$

$\frac{7}{0.1} = 70$

$\frac{7}{0.01} = 700$

$\frac{7}{0.001} = 7000$

$\lim_{n \to 3^+} \frac{7}{n-3} = \infty$

> If $(c, f(c))$ is a point of inflection of the graph of f, then either $f''(c) = 0$ or $f''(c)$ is undefined.

Step by Step Function Plotting

1. Find the formulas for the first and second derivative of the function.
2. Find the critical numbers.
3. Make a tri-columnar table in which the first column contains the intervals defined by the critical numbers and points of discontinuity, the second column contains the sign on the first derivative over each of these intervals, and the third column describes the implied graphical behavior of the function.
4. Find the possible inflection points.
5. Make a tri-columnar table in which the first column contains the intervals defined by the possible inflection points and points of discontinuity for the function, the second column contains the sign on the second derivative over each of these intervals, and the third column describes the implied graphical behavior of the function.
6. Plot the vertical and horizontal intercepts, extreme points and inflection points. Plot any asymptotes and/or holes for the function (use limits to find these).
7. Connect the dots (appropriately).

EXAMPLE: Use the step by step function plotting rules to plot $g(x) = \frac{x(x-4)^2}{x(x+2)}$.

EXAMPLE: Use the step by step function plotting rules to plot $f(x) = 2\sin(x) + \sin^2(x)$ on $[0, 2\pi]$.

$x \neq 0$
$x \neq -2$

$$g(x) = \frac{x(x-4)^2}{x(x+2)}$$ The domain of g is $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$.

$$= \frac{(x-4)^2}{x+2}, \text{ for } x \neq 0$$

$$g'(x) = \frac{(x+2)\frac{d}{dx}\left((x-4)^2\right) - (x-4)^2 \frac{d}{dx}(x+2)}{(x+2)^2}$$

$$= \frac{(x+2)\cdot 2(x-4)^1 \cdot 1 - (x-4)^2 \cdot 1}{(x+2)^2}$$

$$= \frac{(x-4)\left[2(x+2) - (x-4)\right]}{(x+2)^2}$$

$$= \frac{(x-4)(2x+4-x+4)}{(x+2)^2}$$

$$= \frac{(x-4)(x+8)}{(x+2)^2}, \text{ for } x \neq 0$$

$g'(x) = 0$ if $x = 4$ or if $x = -8$
$g'(x)$ is undefined if $x = -2$ or $x = 0$.
The critical numbers of g are -8 and 4.

Table 1: $g'(x) = \frac{(x-4)(x+8)}{(x+2)^2}$, for $x \neq 0$

Intervals	Sign of g'	graphical behavior of g
$(-\infty, -8)$	$\frac{-\cdot -}{+} = +$	increasing
$(-8, -2)$	$\frac{-\cdot +}{+} = -$	decreasing
$(-2, 0)$	$\frac{-\cdot +}{+} = -$	decreasing
$(0, 4)$	$\frac{-\cdot +}{+} = -$	decreasing
$(4, \infty)$	$\frac{+\cdot +}{+} = +$	increasing

The First Derivative test and Table 1 show that g has a local maximum at -8 and a local minimum at 4.

$$g(-8) = \frac{(-8-4)^2}{-8+2} = \frac{(-12)^2}{-6} = \frac{144}{-6} = -24$$

The local maximum of g is -24.

$(-8, -24)$

$$g(4) = \frac{(4-4)^2}{4+2} = 0$$

The local minimum of g is 0.

$(4, 0)$

$$g'(x) = \frac{(x-4)(x+8)}{(x+2)^2}, \text{ for } x \neq 0$$

$$= \frac{x^2+4x-32}{x^2+4x+4}$$

$$g''(x) = \frac{(x^2+4x+4)\frac{d}{dx}(x^2+4x-32) - (x^2+4x-32)\frac{d}{dx}(x^2+4x+4)}{(x^2+4x+4)^2}$$

$$= \frac{(x+2)^2(2x+4) - (x-4)(x+8)(2x+4)}{[(x+2)^2]^2}$$

$$= \frac{(2x+4)[(x+2)^2 - (x-4)(x+8)]}{(x+2)^4}$$

$$= \frac{2(x+2)[x^2+4x+4 - (x^2+4x-32)]}{(x+2)^4}$$

$$= \frac{2(4-(-32))}{(x+2)^3}$$

$$= \frac{2(36)}{(x+2)^3}$$

$$= \frac{72}{(x+2)^3}, \text{ for } x \neq 0$$

$g''(x)$ never equals 0.
$g''(x)$ is undefined at $x=-2$ and at $x=0$, but these are discontinuities of g so g has no inflection points.
Caution! This doesn't mean g never changes concavity. g might change concavity at a discontinuity.

Table 2: $g''(x) = \frac{72}{(x+2)^3}$, for $x \neq 0$

Interval	Sign of g''	graphical behavior of g
$(-\infty, -2)$	$-$	concave down
$(-2, 0)$	$+$	concave up
$(0, \infty)$	$+$	concave up

$$g(x) = \frac{x(x-4)^2}{x(x+2)}$$

Intercepts?

y-intercept

set $x=0$

$g(0)$ is undefined

g has no y-intercept

x-intercept(s)

set $y=0$

$g(x)=0$, if

$x=4$

$(4,0)$ is the only x-intercept

Discontinuities

$x=0$ removable

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{x(x-4)^2}{x(x+2)}$$

$$= \lim_{x\to 0} \frac{(x-4)^2}{x+2}$$

$$= \frac{(0-4)^2}{0+2}$$

$$= \frac{16}{2}$$

$$= 8$$

g has a hole at $(0,8)$

$x = -2$ $\qquad\qquad g(x) = \dfrac{x(x-4)^2}{x(x+2)}$

$$\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} \frac{x(x-4)^2}{x(x+2)}$$

$$= \lim_{x \to -2^+} \frac{(x-4)^2}{x+2}$$

$$= \infty$$

Rational limit form $\dfrac{36}{0^+}$

$$\lim_{x \to -2^-} g(x) = \lim_{x \to -2^-} \frac{(x-4)^2}{x+2}$$

$$= -\infty$$

Rational limit form $\dfrac{36}{0^-}$

g has a vertical asymptote at $x = -2$.

End-behavior (any horizontal asymptotes?)

$$\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{(x-4)^2}{x+2}$$

$$= \lim_{x \to \infty} \frac{x^2 - 8x + 16}{x+2}$$

$$= \lim_{x \to \infty} \frac{x^2}{x}$$

$$= \lim_{x \to \infty} x$$

$$= \infty$$

Look at the dominant term of the numerator and the dominant term of the denominator

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$$\lim_{x \to -\infty} g(x) = \lim_{x \to -\infty} \frac{x^2 - 8x + 16}{x + 2}$$

$$= \lim_{x \to -\infty} \frac{x^2}{x}$$

$$= \lim_{x \to -\infty} x$$

$$= -\infty$$

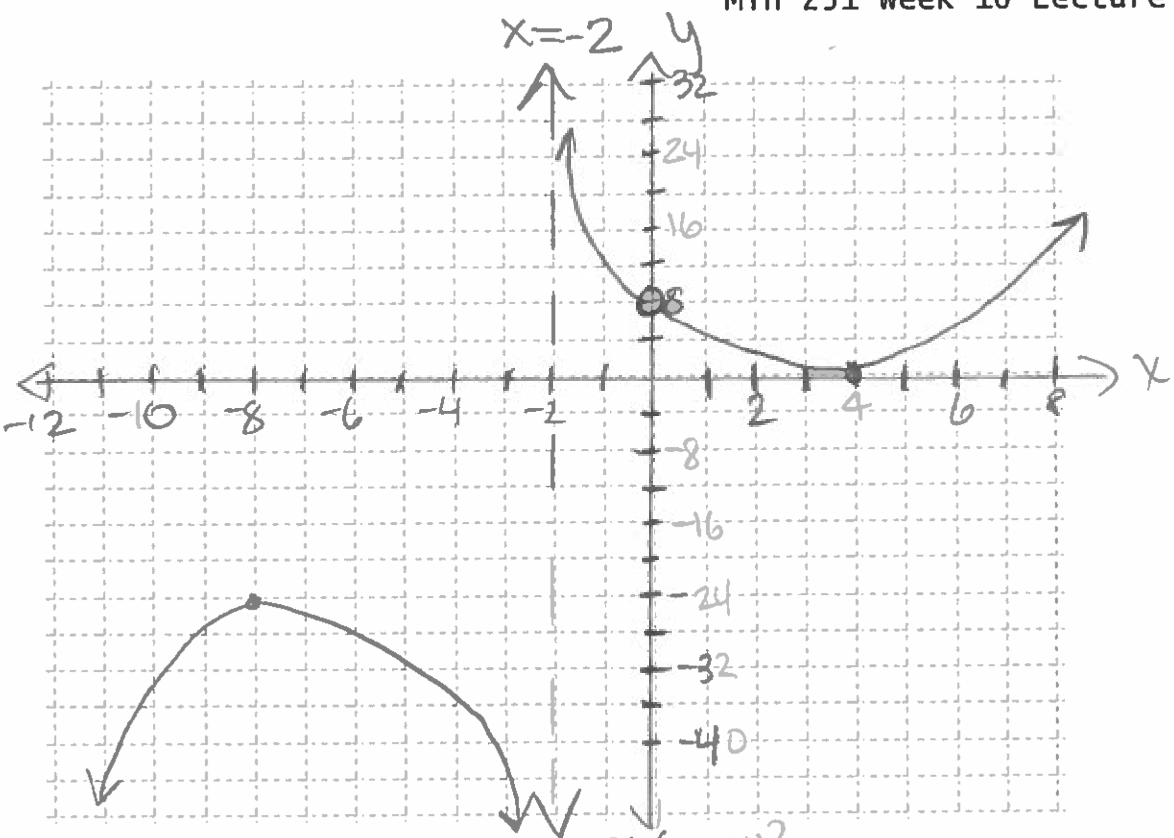

Figure 1: $g(x) = \frac{x(x-4)^2}{x(x+2)}$

max
(-8, -24)

min
(4, 0)

hole
(0, 8)

asymptote
x = -2